TRIGONOMETRICAL

SURVEYING, LEVELLING,

AND

RAILWAY ENGINEERING.

BY

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TO

THE RIGHT HONOURABLE

THE EARL OF HADDINGTON,

FIRST LORD COMMISSIONER OF THE ADMIRALTY,

&c. &c. &c.

My Lord,

I gratefully acknowledge the favour conferred upon me by your permission to inscribe to you the following Treatise on Trigonometrical Surveying. The investigations from which the Formulæ and Rules are derived cost me some mental exertion, and the computation of several of the Tables considerable mechanical labour. I shall, however, feel in some measure compensated for these, should the Surveys of the Officers of the service over which you have been called upon so honourably and advantageously to preside, be, by their means, facilitated and improved.

I have the honour to be,

My Lord,

Your Lordship's most obedient and

Very humble Servant,

WILLIAM GALBRAITH.

PREFACE.

Of late years the art of Surveying has made rapid advances in accuracy and precision, whether in reference to the improvement of instruments, the modes of observation, or the methods of reduction. The Trigonometrical Surveys, by different Governments of Europe, have partaken even somewhat of a national rivalry in the importance of their results, and in the application of the sciences to elicit whatever appeared to be most valuable or instructive.

The processes followed about the end of last century are now, in a great degree, superseded by those that are more accurate, as well as more easy. Many of these, however, can be followed successfully by the mathematician alone, and are totally unintelligible to the ordinary classes of professional men. To remove this difficulty, as far as my limits will permit, has been my object in the present work, in which I have given the results of my investigations only, in the shape of formulæ, rules, and tables.

Those connected with railways will, it is hoped, prove peculiarly useful at the present time, when so many lines are daily projected, whose relative capabilities are so much required to be investigated, both by the engineer and an intelligent public. It has been by tables such as these that the Parliamentary Reports on the relative merits of the London and Edinburgh, and the London and Glasgow competing lines, have been drawn up, and they are indispensable in all similar researches.

Indeed all those tables chiefly useful to the practical man have, for that reason, been rendered full and precise, and their uses are clearly explained by numerous examples; while every encouragement has been given me by the Publishers to extend their utility and ensure their accuracy.

This Treatise will be found, independent of any other work, very complete of itself on the subjects of which it treats. To those, however, requiring information on the practice of Landsurveying, the method of keeping field-books, planning estates, and finishing drawings of almost every variety of ground, Ainslie's Treatise on that subject will be found very satisfactory. To accommodate those professional men who may be so inclined, the Publishers have caused to be printed a limited number of copies of our Treatise in quarto, so as to correspond with Ainslie's work, and, when bound up along with it, the whole will form an extensive body of Surveying in almost all its departments.

EDINBURGH, *December* 1841.

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TRIGONOMETRICAL SURVEYING, &c.

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1. THE figure of the earth is nearly that of a globe, and, for many purposes of surveying, this hypothesis will bring out conclusions sufficiently accurate; but for the nicer and more extended processes, the earth must be considered as a spheroid compressed at the poles. The different measures of arcs of the meridian, &c. concur to prove that the compression is about $\frac{1}{300}$, that is, the polar semiaxis is one-three-hundredth part less than the equatorial radius. From the comparison of a number of arcs, I have found the radius of the equator equal to 20922642 feet, and the polar semiaxis 20852900 feet; and from these, by the properties of the conic sections, the various formulæ, rules, and auxiliary tables required in Trigonometrical Surveying and Levelling are obtained.

2. Though an extensive trigonometrical survey may be commenced by any of its details, yet it is usual to measure a base, in the first instance, with all possible attention to accuracy. It is generally chosen in as level a position as may be attainable, and it is a good plan to measure it first approximately by a hundred-feet chain, as a trial of its capabilities, and a check on the more accurate methods to be afterwards followed. A good theodolite, or transit instrument, is placed securely on a station at one extremity,

and, by the motion of the telescope in a vertical plane, such a number of stakes are intersected throughout the base, by this means placed in a straight line, as are sufficient to guide the subsequent measures. In the course of this process, considerable trouble will be sometimes experienced from the effects of *lateral refraction*, which shifts the stakes sometimes to the right and at other times to the left. The same atmospheric irregularities render it necessary to measure the horizontal and vertical angles repeatedly in the subsequent course of the survey, on different days at the most favourable hours, however powerful the instrument employed may be.

3. Ramsden's steel-chain, made in a peculiar manner, seemed to answer the purpose of lineal measure tolerably well; but it appears that Colonel Colby's compensation bars, constructed by Troughton, and composed of steel and brass, connected on an ingenious plan, possess a decided advantage, because the measurement is not carried on by a contact of the ends as in Roy's glass-rods, or the French metallic rods, with sliding *languettes*, but by ascertaining their coincidence from fine points on platina, with powerful microscopes, having cross wires in their foci, in a manner similar to the coincidence of verniers, or rather to the examination of the divisions of astronomical circles, by powerful reading microscopes. These microscopes are placed on compensating bars also, like the measuring rods, while all these bars themselves have been accurately tested by actual experiment, and found correct. It was in this way that the base-line on the shores of Loch Foyle in Ireland was measured—the most accurate operation of the kind perhaps hitherto performed.

4. From the extremities of this accurately measured base, angles are taken with the theodolite to other properly

selected points, and thence extended over that portion of the country to be surveyed,—the triangles, for the sake of accuracy, being chosen as nearly equilateral as possible.

5. The measurement of an arc of the meridian generally either accompanies, or is derived from, the operations connected with the survey. For this purpose, the position of the meridian, passing through one of the extremities of the base, or some of the angular points of the series of triangles, must be determined by a good theodolite, an astronomical circle, or by one of the best transit instruments. Then the angle which some of the sides of the adjacent triangles makes with the meridian must be accurately measured, from which the bearings of all the sides of the connected series of triangles may be found, in order to obtain either an arc of the meridian or to find the latitudes and longitudes of prominent points in the course of the survey.

The same operations must be repeated for the purpose of verification at the termination of the series, or oftener, if the survey be of great extent.

6. The latitudes of the extremities of the arc, or of two points adjacent and trigonometrically reduced to them, must be determined by the astronomical circle, or other proper instrument, from numerous observations on the same stars, at the same time as nearly as possible, so that any small error in the mean places of the stars, or in the necessary reductions, may be thus avoided.*

7. There are three different methods of making the usual calculations of the sides and angles of the triangles—the first by treating them as spherical triangles; the second by

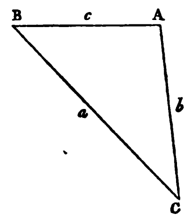
* About the 21st of August 1840, when I observed at Inchkeith, the declination of α Aquilæ, as given in the Connaisance des Tems, exceeded that in the Nautical Almanac by $2\frac{1}{2}''$!!! while that of Polaris agreed nearly.

reducing the angles of the arcs to those of their chords; and the third, the easiest of the three, and sufficiently accurate for every practical purpose, is to deduct one-third part of the *spherical excess*, that is, the excess of the three spherical angles above two right angles, and using the remainders in the calculation, which give the lengths of the opposite sides sensibly the same as that by spherical trigonometry, or by a reduction to the chords, with much less trouble. In this last case, it ought to be recollected that the vertical spherical angles, before deducting one-third of the spherical excess, are equal; but often they may be unequal, if the triangles to which they respectively belong be unequal, since the spherical excess is proportional to the magnitude of the triangle. The angles so deduced are, for the sake of distinction, called *mean angles.*

8. To estimate the corrections to be applied to horizontal angles, measured on the surface of the earth at any point of observation, let m be the arithmetical mean of the whole, and the seconds of reading s, s', s'', &c. and rejecting from each observation the same quantity, giving the results, if more convenient, a negative sign; then $m-s$, $m-s'$, $m-s''$, &c. are the differences of the individual observations from the mean, and the *weight* of the determination, as it is technically called, or of the average m, is equal to the square of the number of observations divided by twice the sum of the squares of the errors, as shewn in the usual treatises on probabilities. In this manner the weight is found for each angle, and the error of the three angles of the triangle is the difference between the sum of the three angles, of which each is the mean of the observed angles, and $180° + \epsilon$ is divided into three parts proportional to the reciprocal of the weights, which parts form the corrections to be applied, according to their signs, to the angles to which they respec-

tively belong. We have then the three corrected spherical angles, the sum of which is exactly $180° + \epsilon$, in which ϵ is the small quantity called the spherical excess.

9. Example 1. Let A be East Lomond in Fifeshire, B Ben Cleuch in the Ochils, and C the Calton Hill at Edinburgh.



Observed Angles.

	°	′	″	
A =	83	55	46.06	by 7 observations.
B =	45	38	55.19	by 6 observations.
C =	50	25	15.83	by 20 observations.
Sum =	179	59	57.08	
$180 + \epsilon$ =	180	0	2.79	
Error			5.71	

Though the preceding method (§ 8) be more strictly scientific, yet for ordinary purposes this error may be distributed among the angles simply as the reciprocal of the number of observations, thus;—

$\frac{1}{7} = 0.143$; and $0.360 : 5''.71 :: 0.143$: correction of A $+2''.27$

$\frac{1}{6} = 0.167$ 0.167 : correction of B $+2.65$

$\frac{1}{20} = 0.050$ 0.050 : correction of C $+0.79$

0.360 The whole correction $+5.71$

Hence

		°	′	″	
A	=	83	55	48.33	corrected.
B	=	45	38	57.84	
C	=	50	25	16.62	
		180	0	2.79	

Now, if from each of these one-third of ϵ, or one-third of $2''.79$ be subtracted, there will remain for the *mean angles*

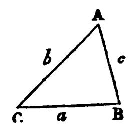
		°	′	″
A	=	83	55	47.40
B	=	45	38	56.91
C	=	50	25	15.69
Sum $- \epsilon$	=	180	0	0.00

Also the length of the arc in feet opposite the angle A, is 146335.0.

1. As	sin A	83° 55′ 47″.40	....	9.9975581
Is to	sin B	45 38 56.91	....	9.8543501
So is	a	146335.0 feet	...	5.1653483
To	b	105230.2 feet	...	5.0221403

2. As	sin A	83° 55′ 47″.40	....	9.9975581
Is to	sin C	50 25 15.69	....	9.8869119
So is	a	146335.0 feet	...	5.1653483
To	c	113423.3 feet	...	5.0547021

almost as exact as the more complex method.



Ex. 2. Let A be Benlomond in Stirlingshire, B Cairnsmuir upon Deugh in Galloway, and C Knocklayd in Antrim in Ireland, we shall have, by the more complex method (§ 8) of distributing the errors,

Observed Angles.

A	=	56° 43′ 28″.58	by 3 observations.
B	=	79 42 28.69	by 1 observation.
C	=	43 34 36.89	by 2 observations.
Sum	=	180 0 34.16	

A by 1st obs.	56° 43′ 29″.97	s
2d ...	27.04	s'
3d ...	28.72	s''
m =	56 43 28.58	

Hence

$$m-s=+1''.39,\quad (m-s)^2=1''.9321 \qquad N=3$$
$$m-s'=-1.54,\quad (m-s')^2=2.3716 \qquad N^2=9$$
$$m-s''=+0.14,\quad (m-s'')^2=0.0196$$
$$S^2= \ldots\ldots\ 4.3233$$

$\frac{N^2}{2\,S^2}=\frac{9}{8.6466}=1.041=$ weight, and $\frac{2\,S^2}{N^2}=0.9607=$ the reciprocal of the weight. In like manner

C 1st obs.	43° 34′ 38″.36	s	
2d ...	35.43	s'	
m =	43 34 36.895		
$m-s$ = +	1.465,	$(m-s)^2 = 2.1462,$	N = 2
$m-s'$ = −	1.465,	$(m-s')^2 = 2.1462,$	$N^2 = 4$
S^2 =	. . . . .	4.2924	

$\frac{N^2}{2\,S^2} = 0.4660 =$ weight. Reciprocal $\frac{2\,S^2}{N^2} = 2.1462$. There being only one observation of the angle B, its weight cannot be computed like those of the other angles. Its weight must either be assumed or estimated by a comparison with those of the other angles. As the reciprocals of the weights in the other two angles are inversely as the squares of the number of observations nearly, this may also be estimated in the same ratio, and $\frac{2\,S^2}{N^2} = 8.6155$ nearly. Whence the sum $= 0.9607 + 2.1462 + 8.6155 = 11.7224 = \Sigma\varrho$, according to which the error must be applied by distributive proportion as in last example.

The spherical excess must now be computed by the formula

$$\epsilon'' = \frac{R''\,a}{r^2} = F^2\,a \sin 1'' \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

in which a is the area of the triangle in square feet, F the factor from Table XIX. to convert feet into arcs. If the mean radius of curvature of the earth be taken, which, for moderate triangles will be sufficient, formula (1) becomes

$$\epsilon'' = \frac{a}{2122300000} \text{ nearly} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

The log of 2122300000 is 9.3268079, and its arithmetical complement is 0.6731921, a constant log, to which the log a, the log of the area of the triangle, being added, will give the log of ϵ', the spherical excess in seconds to be applied as formerly indicated.

Whence the spherical excess amounts to 1″ in about 76 square miles.

From this an easy rule may be derived to find the spherical excess by a simple calculation, or even by the common sliding rule from a plan of the triangles, to which a scale of miles is adapted for measuring the base and perpendiculars in an approximate manner.

Set 152 on the sliding line of numbers to the base of the triangle in miles, then opposite to the perpendicular will be found the spherical excess in seconds and decimals, true to nearly two places of decimals in moderate triangles.

The triangle now under consideration being large, the more accurate formula (1.) will be employed to find ϵ''.

To mean latitude of the triangle about 55° N., and azimuth 45°, there will be found, by the aid of Table XIX. &c.

Log $\frac{1}{2}$ sin 1″	4.384545
2 Log F (Table XIX.)	5.986630
B = 79° 42′ 28″.7, sin	9.992955
side a = 426794 feet, log	5.630402
side c = 352038 feet, log	5.546589
ϵ'' = 34″.763, log	1.541121
$\frac{1}{3}\epsilon''$ = 11 .588	
By calculation ϵ is	34″.763
By observation it is	34.160
Error of observation	0.603

to be distributed among the observed angles in the ratio of the reciprocals of their respective weights.

As 11.7224 :	$-0''.603$	: : 0.9607 :	$+0''.050$
		: : 2.1462 :	$+0.111$
		: : 8.6155 :	$+0.442$
Sum = ...			0.603

Hence the following spherical angles will be obtained,

A = 56° 43′ 28″.58 + 0.05 =	56° 43′ 28″.63	
B = 79 42 28.69 + 0.44 =	79 42 29.13	
C = 43 34 36.89 + 0.11 =	43 34 37.00	
Corrected sum = 180° + ϵ =	180 0 34.76	

It must therefore be remembered that each of these angles is equal to its opposite vertical angle, and not those diminished by the effects of the spherical excess which immediately follow.

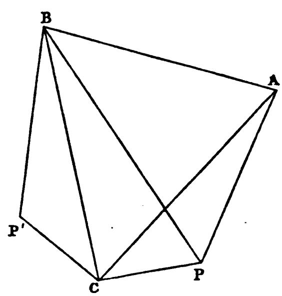
If, from each of the spherical angles thus determined, one-third of the spherical excess be deducted, the remainders will be the mean angles.

A = 56° 43′ 28″.63 − 11″.59 =	56° 43′ 17″.04
B = 79 42 29.13 − 11.59 =	79 42 17.54
C = 43 34 37.00 − 11.58 =	43 34 25.42
Sum . . .	180 0 0.00

Log arc c in feet = log 352037.62	.	5.5465891	α
Log sin A 56° 43′ 17″.04	.	9.9222127	β
Log sin B 79 42 17 .54	.	9.9929511	γ
a . c . log sin c 43 34 25 .42	.	0.1615997	δ
Log arc a 426974.06 feet .	.	5.6304015	$\alpha+\beta+\delta$
Log arc b 502504.47 feet .	.	5.7011399	$\alpha+\gamma+\delta$

In the Trigonometrical Survey, this operation is performed by reducing the spherical angles to those of the corresponding chords, which, in this large triangle, would give precisely the same results. That method requires considerably more labour without almost any corresponding advantage, and is now very generally abandoned. Since the observations by which the angles in Example 1. have been determined were about *six* times more numerous than those in the second, while the error in the former is nearly *ten* times greater than that in the latter, it seems that the laborious calculations depending upon the doctrine of probabilities in such cases may be very well saved, and that the method of distributing the errors proportionally to the reciprocals of the number of observations, as in the first case, is fully sufficient. In the present case, the mean angles would be A=56° 43′ 17″.10, B=79° 42′ 17″.43, and C=43° 34′ 25″.47, which would give results not differing much from the preceding.

10. The centre of the instrument should always, when possible, be placed in the vertical line occupied by the axis of the signal. When, however, this cannot be conveniently done, the observed angles must be reduced to it by an appropriate formula.



Let APB be the observed angle to be reduced to ACB, that at the axis of the signal C.

For this purpose it is necessary to measure the distance CP. Let the angle APB=P, BPC=p, the angle of direction reckoned from the observed object on the left to the axis of the signal, CP=d, AC=r the distance to the *right*, and BC=l the distance to the *left*,

Then $$C-P=R''d\left\{\frac{\sin(P+p)}{r}-\frac{\sin p}{l}\right\} \quad . \quad . \quad . \quad (3.)$$

or $$C-P=R''d\sin P\sin(A-p)\div r\sin A \quad . \quad . \quad (4.)$$

in which R″ is the arc 206264″.8, the arc equal to the radius in seconds. When the theodolite cannot be conveniently placed at the same height as the top of the signal observed, the correction of the zenith distance will be

$$d\delta=\frac{R''\,dh\sin\delta}{D}=\frac{R''\,dh}{D}\text{ nearly} \quad . \quad . \quad . \quad (5.)$$

when δ differs little from 90°, in which δ is the observed zenith distance, dh the difference between the height of the centre of the circle and the point observed, and D the distance. In these formulæ the signs of the trigonometrical quantities must be carefully attended to.

EXAMPLE 1. Let P=65° 41′ 6″.5, p=181° 35′ 13″.5, d=155 feet, r=33329.8 feet, and l=74707.5 feet; required the reduction of P to C by formula (3)?

Log R″ . . .	5.314425		
Log d . . .	2.190332		
	+7.504757	. . .	−7.504757
P+p=247° 16′ 20″ sin	−9.964896	p=181° 35′ 13″.5 sin	−8.442422
a . c . log r . .	+5.477167	a . c . log l .	+5.126636
1st time −884″.8 log	−2.946820	2d time +11″.8 log	+1.073815
2d time + 11 .8			

−873 .0=−14′ 33″.0
P = . . 65″ 41 6 .5

C = . . 65 26 33 .5 corrected.

Ex. 2. From Allington Knoll the staff on Tenterden Steeple had a depression of 3′ 51″.0, or δ=90° 3′ 51″.0, and the top of the staff was 3.1 feet higher than the axis of the instrument when at that station. On Tenterden Steeple the ground at Allington Knoll was depressed 3′ 35″.0, or δ'=90° 3′ 35″.0, and the axis of the instrument when at this station was 5.5 feet above the ground; required the corrections of the observed zenith distances, the lineal distance between the stations being 61777.5 feet?

Log R″ . .	5.314425	. . .	5.314425
a . c . log D .	5.209169	. . .	5.209169
dh= +3.1 feet log	+0.491362	dh= −5.5 feet log	−0.740363
$d\delta$= +10″.35 log	+1.014956	$d\delta'$= −18″.36 log	−1.263957

the corrections of the zenith distances sufficiently accurate by the more simple formula, since δ and δ' are so near 90°.

When these corrections are to be applied to the angles between the verticals of two given points, they may be combined as follows:—

Log R″	. . . .	5.314425
$dh + dh' = +3.1 - 5.5 = -2.4$ feet log		-0.380211
a . c . log D	. . . . .	5.209169
$s + s' = -8''.02$ log	. . . .	-0.903805

the same as $d\delta + d\delta' = +10''.35 - 18''.36 = -8''.01$.

(11.) In measuring horizontal or vertical angles in reference to terrestrial objects, if the atmosphere is not sufficiently clear, it is difficult to intersect the signals with the necessary accuracy. In this case an instrument called a heliotrope is generally used to reflect the sun's image in the direction of the observer. I have found the usual reflecting horizon of coloured glass set in a frame, turning on a horizontal and on a vertical axis to obtain any requisite inclination, very convenient for this purpose. The proper direction of the sun's image may be given by a circular piece of polished block tin or brass, with a circular hole of three or four inches in diameter in it, stuck in the groove of the usual offset-staff, through which hole the station of the observer must be seen, while, by reflection from the glass, the ring of the perforated disk must be illuminated.

12. To conduct a general series of observations, either on land or at sea, for the purposes of surveying, &c. the following general remarks will be found useful.

1°, To record the state of the barometer and thermometer, three or four times a-day, more especially when making observations.

2°, To take altitudes carefully on objects useful for time, &c. from three to six hours distant from the meridian.

3°, To find the error of chronometer as often as possible, to be able to compute the correct time of transit of the sun, stars, &c. by it, for latitude by circum-meridian altitudes, &c.

4°, To observe objects having equal altitudes nearly, to the north and south of the zenith, to destroy the effects of errors in the instruments employed, such as bias of axis, errors of division, glasses, artificial horizon, &c.; the same method should be pursued for the accurate determination of time by selecting objects to the east and west.

5°, In the case of marine surveys, to observe on land as often as safe and convenient, with the best instruments for time, latitude and longitude, by lunars, moon-culminating stars, occultations, &c.

6°, To choose a station somewhat elevated, free from woods, jungle, &c. so that, with ordinary care, surprise by the natives will be impossible or difficult.

7°, To take magnetic bearings of well-defined and conspicuous objects whenever practicable, from points well determined in latitude and longitude. If convenient, angular observations with the theodolite and other instruments would be better.

8°, To repeat your observations if possible at least *three* times, to guard against mistakes, which, even with the greatest care and experience, will sometimes happen. To make one or more assistants take observations along with you, and to receive their reports without communicating your own. If there be such a difference as to indicate a decided fault somewhere, the observations ought to be repeated till the cause of the discrepancy be removed.

9°, To make such calculations only as may be absolutely necessary to carry on a connected series of useful observations.

10°, To keep regular and clearly written note-books, on a systematic plan, in which every thing is recorded, so that any mathematician or astronomer may be enabled to deduce fair conclusions at any future period. These books must be all properly ruled, titled, and numbered, for future reference. Marks and abbreviations should be all carefully recorded and explained.

13. These general views being premised, it will now be necessary to enter into the practical details. It is hardly possible, in these operations, to divest the formulæ entirely of an algebraical character in some cases, though it will be done as often as possible. One of the processes in trigonometrical surveying is the determination of the latitude. This operation is most simply performed by a meridian altitude or a zenith distance, and if a circumpolar star be selected, the result will be independent of the exact position of the star, because the latitude, in that case, is equal to half the sum of the altitudes of the star above and below the pole, corrected for the effects of refraction by Table V. When a zenith sector like that belonging to the Board of Ordnance is used, the stars must be selected near the zenith, and consequently little error is to be feared from the effects of refraction, while the great power of its telescope, and the general accuracy of its construction, render a single observation by it a close approximation to the truth. When, however, the smaller classes of instruments are employed, it then becomes necessary to repeat the observations near the meridian, reducing those taken at a short distance, such as about ten or fifteen minutes, to what they would have been on it, from a knowledge of its distance from that circle in time, the approximate latitude and declination of the object observed. In this way the results from smaller instruments become nearly equivalent to those of the greater, since as many observations may be taken by the former in one day as by the latter in ten.

ON FINDING THE LATITUDE.

14. The most easy and ready way of finding the latitude

is by a meridian altitude of a celestial body whose declination is known. Should the object have a sensible diameter, like the sun or moon, the altitude or zenith-distance of the lower or upper limb, or, what is superior, both are alternately observed, and, by the application of several corrections, that of the centre is obtained.

When reflecting instruments, such as the sextant, repeating circle, &c. with an artificial horizon, are employed, the arc read off must, from the nature of the instruments, be halved before the corrections are applied. At sea, since the lower limb of the sun, moon, or the centre of a star, is generally brought to the visible horizon, the dip from Table I. must be subtracted before the corrections from Table II. &c. are taken. At land, a meridian altitude of the sun, moon, or a planet, must be corrected for refraction, parallax, and semidiameter, but not for dip. At sea, the same corrections are applied after the dip has been subtracted. All these may be found by the following tables and the Nautical Almanack. The refraction constitutes the whole correction of a fixed star at land. At sea, the dip must be previously subtracted.* If the instrument does not give the zenith distance, it may be found by taking its complement to 90°, denominated *north* or *south*, according as the observer is north or south of the object.

Now, if the zenith distance and declination be of the *same name*, their *sum* is the latitude; but if of *contrary names*, their *difference* is the latitude, of the same name as the greater.

Example 1. At Pladda Light, in longitude 20^m 30^s W., 20 feet above the level of the sea, on the 15th of August 1836, the follow-

* The method of applying all these corrections is given in the explanation of the tables, and illustrated by the following examples.

ing observations of the sun's lower limb referred to the sea-horizon, were made with a pocket-sextant, within two or three minutes of the meridian, and both sides of it in succession; required the latitude?

1st observation, sun's lower limb,	48° 20′ 0″
2d ...	21 0
3d ...	21 0
4th ...	20 0
Mean of the four	48 20 30
Dip to 20 feet (Table I.)	− 2 24
	48 18 6
Correction to alt. 48° (Table II.)	− 0 46
True alt. of sun's lower limb	48 17 20
Sun's semidiameter by Nat. Almanac	+ 15 49
True alt. of sun's centre	48 33 9
	90 0 0
True zenith-distance	41 26 51 N.
Sun's declination by Nat. Almanac*	13 59 5 N.
True observed latitude	54 25 56 N.

2. At Edinburgh, on the 13th of March 1841, the following observations were made with a Dollond's sextant and an artificial horizon, one-half of which was made by a contact of the lower limbs, and the other by a contact of the upper, alternately, while the artificial horizon was reversed at the middle of the observations.

	h.	m.	s.
Time of apparent noon	12	0	0
Equation of time at Edinburgh	+	9	41
Error of watch	±	0	0
Time of transit by watch	12	9	41

In regard to reading, when the zenith-distance does not exceed 90°, I have caused on a pocket-sextant be engraved numbers on the arc, commencing with 0° at 90°, in an order

* See explanation of Table XXX.

the reverse of that usually adopted, and likewise on the vernier, so that I read zenith-distances in place of altitudes, even with the sextant, in such cases as it may appear more convenient, or I may read alternately, in different series of observations, both ways, as a check upon each other, to avoid mistakes in the reading. In the present instance, however, the zenith-distance, when doubled, exceeded the limits of the instrument, as with the artificial horizon must be the case, and therefore double altitudes were necessarily taken.

Barometer 30.3 *inches.* *Thermometer* 53° *Fahr.*

	Times of Observation.		Double Altitudes.
	h. m. s.		° ′ ″
1st observation .	11 58 35	. .	61 42 50 l. l.
2d	12 1 53	. .	62 59 0 u. l.
3d	12 7 12	. .	62 53 30 u. l.
4th	12 9 58	. .	61 56 50 l. l.
5th	12 15 16	. .	62 52 30 u. l.
6th	12 19 20	. .	61 52 40 l. l.
Means . .	12 8 42.3	. .	62 22 53.3
Long. W. . .	+ 12 43.5,	Half .	31 11 26.7
Error of watch .	0 0.0	. .	90 0 0.0
Greenwich M. T.	12 21 25.8	Zenith-dist.	58 48 33.3 N.

The refraction must now be computed by Table V.:

Zenith-distance observed 58° 48′.6, log $\delta\,\theta$	.	1.9832
Barometer $b = 30^{in}.3$, log (Table VI.)	.	0.0043
Thermometer $\tau = 53°$, log (Table VII.)	.	9.9999
Thermometer $t = 53°$, log (Table VIII.)	.	9.9973
Refraction . . . $r = 96''.5$ log	.	1.9847
Sun's parallax (Table XII.) $= -7\ .4$		

$r - \pi =$ cor . .	89 .1 =	0° 1′ 29″.1
Mean zenith-distance . . .		58 48 33 .3 N.
Corrected zenith-distance . . .		58 50 2 .4 N.

It is now necessary to apply the reduction of the different particular observations by Table XVII., to reduce each to what it would have been had it been made precisely on the meridian, which is most concisely done by grouping the whole together. For this purpose let Δ be the required zenith-distance upon the meridian, and δ that obtained as above, then, in order to reduce δ to Δ, we have the following formula :—*

$$\Delta = \delta - 2\sin^2 \tfrac{1}{2} t \cos l \cos d \operatorname{cosec}(l-d) + 2\sin^4 \tfrac{1}{2} t \{\cos l \cos d \operatorname{cosec}(l-d)\}^2 \cot(l-d) \quad . \quad (6.)$$

in which t is the time from the meridian either before or after transit, in mean solar time if the sun be observed, but in sidereal if a star, l the latitude, and d the declination, reckoned *minus* if of a contrary name to l. This distinction may be avoided by substituting the zenith-distance for $d-l$. It is clear that $2\sin^2 \frac{1}{2} t$ is the versine of t, and that $2\sin^4 \frac{1}{2} t$ is half the square of the former, which are designated V and v in the table. To express the reduction in seconds of arc, each of these must be multiplied by R″, an arc equal to the radius in seconds. This is accomplished by the logarithms for V and v at the termination of the table, which include also the division of the sum of the versines by the number of observations, thus simplifying the operation considerably. The computation is performed in the following manner :—

	h. m. s.		m. s.	V	v
Transit by watch	12 9 41				
1st observation	11 58 35,	$t =$	11 6	11726	1375
2d ...	12 1 53,	$t_1=$	7 48	5791	335
3d ...	12 7 12,	$t_2=$	2 29	587	4
4th ...	12 9 58,	$t_3=$	0 17	8	0
5th ...	12 15 16,	$t_4=$	5 35	2967	88
6th ...	12 19 20,	$t_5=$	9 39	8863	786
				29942	2588

* This formula is easily deduced from elementary investigations, but we are restricted to practice here.

Now by the formula,

Estimated lat.	55° 56′ 58″N.	cos	9.748129			
Sun's declinat.	2 51 54 S.	cos	9.999457			
Zenith-dist.	58 48 52 N.	cosec	0.067783	cot	9.781954	log f.
Log F		.	9.815369	× 2 =	9.630738	
To 6 observations log for V		.	7.536274	for v	5.235244	
V = 29942 log . . .		.	4.476281	log v	3.412964	
1st term .	−67″.293	log	1.827924	log 2d	8.060900	
2d term .	+ 0 .012					

Reduction . = −67 .281 = . .	− 0° 1′ 7″.3	
Corrected zenith-distance . . .	58 50 2 .4	N.
True meridian zenith-distance . .	58 48 55 .1	N.
Sun's declination	2 51 53 .6	S.
True latitude	55 56 55 .5	N.

By repeating the observations on stars both to the north and south of the zenith, the latitude will be accurately determined.

If the observations for latitude are taken by the mural or transit circle placed truly in the meridian, these are made when the celestial body is in or very near the centre of the field of view, at the intersection of the horizontal and vertical wires; but when the observations are *repeated* near the meridian, an exact knowledge of the time or error of the watch becomes indispensable, in order to find the time of transit by that watch with which the observations are recorded. For this purpose an approximate value of the latitude may be found as shewn in the first example, from which, and the following rule, the error of the watch within a few seconds may be obtained. With this error and a good sextant a nearer approximation to the true latitude may be found as in Example 2. Whence a new determination of the time may be found sufficiently exact to obtain the latitude correctly, if a series of observations, at nearly equal distances from the meridian before and after transit, be employed. The time may also be found by the method of equal altitudes, as shewn in the explanation of Table XVIII, whenever the weather is steady, especially in fine climates. In our unsteady climate, absolute altitudes taken at nearly equal distances from the meridian east and west, and as close upon the prime vertical as possible, will prove very satisfactory, and then corresponding observations will not easily be lost.

TO FIND THE TIME.

15. Set down under each other the true altitude, polar distance, and latitude. Find half the sum of these three, and the difference between that half sum and the altitude. Then to the log cosecant of the polar distance add the log secant of the latitude, the log cosine of the half sum and the log sine of the difference, half the sum of these four logarithms will be the log sine of half the hour angle from the meridian. In case of determining the time in this manner, it would be convenient to estimate it according to the astronomical method of reckoning, namely, from noon to noon throughout the twenty-four hours. Hence in the forenoon of the civil day, the hour angle thus found must be deducted from 24 hours, and the remainder will be the time past noon of the preceding day.

In using a table of reduced versines, such as that given in my collection of mathematical and astronomical tables, the sum of the four logarithms mentioned above, rejecting tens in the index, will be the hour angle to be taken from the top of the page when the observation is made in the afternoon of the given day, but from the bottom if in the forenoon, to give the time past the preceding noon. This result will be the apparent solar time, to which the equation of time reduced, for the approximate time and longitude, to the corresponding Greenwich mean time (G. M. T.), according to the directions given in the Nautical Almanac (N. A.), page 1 of each month, will give the mean solar time (M. T.) at the place of observation.

If a star be the object, the horary angle must be taken from the top of the page if the star be west of the meridian, but from the bottom if east, to be *always* reckoned west (W.). To this meridian distance add the star's right ascension reduced to the given time, and the complement to 24 hours of the sidereal time at mean noon (S. T. M. N.), reduced by Table XXVI. to the time and place of observation, the sum, rejecting 24 hours as often as possible, will be the mean solar time. If two stars be chosen, one to the east and another to the west, having the same altitudes nearly, any error from a faulty method of observing, or a bias in the instrument, will be avoided, and they should not have more than 30° of declination, because, from their slow motion even on the prime vertical, stars having great declinations are in this case to be avoided.

The method of observing with a sextant having been already shewn, that by the smaller classes of astronomical circles will now be exemplified. That which I generally use is six inches in diameter, having three verniers, each reading 10″, and the scale of its level, a fixed one, indicates 2″ for each division, and reads from a central zero. The general formula to correct for the readings of the level, when applied to the zenith-distance, is

$$l = \frac{(e-o)\,a''}{2\,n} \quad . \quad . \quad . \quad . \quad (7.)$$

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The method of [illegible] ready shewn [illegible] circles will now be [illegible] use is six inches in diameter [illegible] reading 10″, and [illegible] 2″ for each division [illegible] general formula [illegible] when applied to the [illegible] in which l is the resulting effect, e the sum of the readings at the eye-end of the telescope, o the sum of those at the object-end, a'' the value of one division of the scale of the level, and n the number of observations.

In making $a''=2''$ the preceding formula becomes

$$l=\frac{e-o}{n} \quad . \quad . \quad . \quad . \qquad (8.)$$

by the scale of my circle.

In all cases care must be taken of the sign, according to the rules of algebra. The signs must be changed when the instrument shews altitudes. There are three parallel horizontal wires in the focus of the telescope, at each of which the contact of the sun's limb may be observed. I generally observe the contact of the upper limb only at all the three when the sun is ascending, and then, on reversing the circle, the lower limb. I reverse this order when descending, taking care of the apparent change of position, by an astronomical telescope, which shews objects inverted, that is, I observe the *apparent lower limb* first when the object is *ascending*, the apparent upper when descending, consequently the contacts are observed at nearly the same altitude, and have the same refraction.

Examples.—1. On the 11th of August 1836, at Lamlash, in the Island of Arran, in latitude, by estimation, 55° 31′ 56″ N., longitude 20^m 32^s W., the following observations on the sun were made to determine the time. The assistant-watch, by which the observations were made, was 28^s fast of the chronometer, while the barometer stood at 30 inches, and Fahrenheit's thermometer at 50°.

Note.—In the following observations the scale of the level read to 3″ at Lamlash, and formula (7) was employed to find l, the effects of the level; but at Inchkeith it read to 2″, and formula (8) was employed. See pages 23 and 31.

Times by Watch, A. M.		Ver.	Z. D.	Level. + e	Level. — o
	h. m. s.		° ′ ″		
	9 1 49	A	54 42 50	8.5	21.5
		B	43 0		
		C	43 20		
	9 7 7	A	53 35 30	22.0	7.5
		B	36 0		
		C	35 30		
	9 13 52	A	52 46 30	30.0 —	1.5
		B	46 0		
		C	45 20		
	9 17 46	A	52 45 0	15.0	14.0
Mean .	9 10 8.5	B	45 10	75.5	41.5
Watch fast	— 9 43.5	C	45 40	41.5	
Long.	+ 20 32.0 W.				
			53 27 29.2	34.0	
E. G. M. T.	9 20 57.0, *l*	=	+ 12.8	3	
			53 27 42.0	8)102	
	r	=	+ 1 18.7		
	π	=	— 0 6.7	12.75	

True zenith-distance	53 28 54.0		
	90		
True altitude A . .	36 31 6		
Polar distance . .	74 45 9	cosec	0.015563
Latitude	55 31 56	sec	0.247228
Sum	166 48 11		
Half H	83 24 5.5	cos	9.060360
Difference H — A =	46 52 59.5	sin	9.863300
	20 55 23.7	v. s.	9.186451
Equation of time . .	+ 4 54.3		
Mean time	21 0 18.0		

	h. m. s.
Mean time	21 0 18.0
Time by watch + 12^h = . .	21 10 8.5
Watch fast of M. T. . . .	9 50.5
Watch fast of chronometer .	— 28.0
Chronometer fast	9 22.5

On the evening of the same day, by a watch 10^s slow of the same chronometer, and which was gaining 3ˢ a-day, the following observations were made on α Aquilæ.

	Times by Watch.	V.	Z. D.			Level. +	Level. —
	h. m. s.		°	′	″	*e*	*o*
1.	10 18 10	A	47	9	50	22	10
		B		9	20		
		C		9	40		
2.	10 25 50	A	47	3	10	15	16
		B		3	10		
		C		3	30		
3.	10 33 0	A	47	6	20	18	13.5
		B		5	50		
		C		6	20		
4.	10 39 50	A	47	5	0	12	19.0
Mean . .	10 29 12.5	B		5	10	67	58.5
Watch fast	— 9 12.5	C		5	10	58.5	
Long. . .	+ 20 32.0 W.		47	6	2.5	8.5	
		l=	+		3.2	3	
G. M. T. .	10 40 32		47	6	5.7	8)25.5	
		r	+	1	2.8	+3.2	

Corrected zenith-distance = 47 7 8.5 N.

To find the mean time of transit by Tables XXVI. and XXVII., we have, by the Nautical Almanac, the

		h.	m.	s.
Sidereal time at Greenwich mean noon,	σ=	9	19	55.09
Reduction for long. 20ᵐ 32ˢ W. (Table XXVI.)		+		3.37
Sidereal time at Lamlash M. N. . . .		9	19	58.46
Star's right ascension . . .	s=	19	42	49.27
Difference, or $s-\sigma$		10	22	50.81
Reduction to $s-\sigma$ (Table XXVII.) .		—	1	42.04
Mean time of transit		10	21	8.77
Error of watch, fast		+	9	12.50
Time of transit by watch . .		10	30	21.27

	h. m. s.		m. s.
Transit by watch .	10 30 21		
1st observation .	10 18 10,	t =	12 11
2d	10 25 50,	t_1 =	4 31
3d	10 33 0,	t_2 =	2 39
4th	10 39 50,	t^3 =	9 29

$d = 8° 26' 29''.95$ N.

	m. s.	V.	v	For Rate,	
t =	12 11 .	14126 .	1995	M. So. to sid. T. log	+ 0.002375
t_1 =	4 31 .	1942 .	38	Daily Rate,	
t_2 =	2 39 .	668 .	5	Gaining 3^s cor. .	— 30
t_3 =	9 29 .	8560 .	733		
				Log for rate .	+ 0.002345
	Sums,	25296	2771		

Latitude . .	55° 31′ 56″ N.	cos 9.752772	
Declination .	8 26 30 N.	cos 9.995269	
Zenith-dist.	δ = 47 5 26	cosec 0.135233	cot 9.968280 } log f
Log F		9.883274 × 2	= 9.766548 }
Log V′ for 4 . . .		7.712365	log v_1 = 5.411335
V = 25296 log . . .		4.403052	log v_2 = 3.442636
Log for rate . . .		0.002345	log
			8.588799
1st term — 100″.240 log .		2.001036	
2d term + 0 .039			

100. 201 =	— 0° 1′ 40″.20
Corrected zenith-distance	47 7 8 .50
True meridian zenith-distance . .	47 5 28 .30 N.
Star's declination	8 26 29 .95 N.
True latitude, ...	55 31 58 .25 N.

In this manner the observations may be repeated a sufficient number of times to ensure, from a mean of the whole, the requisite accuracy.

When the observations on *stars* are continued for a length of time, the logs of F and f remain nearly constant for the same star, and consequently these may be computed for such a number of stars as may be selected for observation. Indeed, special tables may be drawn up to every ten seconds, and these may be interpolated to every second in t, as was done by myself for α Aquilæ, when I observed at Inchkeith, from which the reduction to the meridian may be made at sight. For this computation special tables are sometimes given, but it may be easily effected by a table of reduced versines employed in the computation of time in last example, or by a table entitled *Rising* in the usual books of navigation.

Let $F''=2\,R''\cos l\cos d\,\text{cosec}\,\delta$ and . . . . . (9.)

$f''=2\,R''(\cos l\cos d\,\text{cosec}\,\delta)^2\cot\delta$ . . . . (10.)

be computed for the given star.

For *a* Aquilæ, at Inchkeith, in latitude 56° 2′ N., in August 1840, when the star's declination was 8° 27′ 7″ N., will be found.

Log F″=	5.489705	log *f* .	5.324769
For $t=15^m$ log R. V. S.	7.029602	× 2 =	4.059204
1st term = −330″.60 log	2.519307		9.383973
2d term = + 0 .24			
Red. = −330 .36			

Continuing this process for every 10^s of *t*, the reduction may easily be found for single seconds by interpolation, which renders this method very easy; and then the smaller classes of circles become in effect nearly equal to the larger, on account of the facility with which observations may be very numerously repeated.

16. In mean latitudes, such as in Britain, observations on the pole-star are very advantageous and convenient for the determination of both latitudes and azimuths at the same time, which may be computed by the following formulæ :—

1. $\sin u = \cos t\tan p$
2. $\sin\lambda = \cos u\cos\delta\sec p$
3. $l = \lambda \pm u$
4. $\sin r = \sin t\sin p$
5. $\tan m = \tan r\sec\lambda$, or
6. $\tan m = \sin t\tan p\sec\lambda$

nearly, and more simply than by (4) and (5) combined.

In these formulæ, t is the sidereal time after transit, p the star's polar distance, λ the latitude of the foot of the perpendicular arc from the star upon the meridian, and δ the zenith distance. Also l is the true latitude, in determining which u is *minus* in the first and fourth quadrants of t, and *plus* in the second and third. In like manner r is the perpendicular from the star upon the meridian, and m the azimuth.

17. If the latitude be previously well known, the azimuth may be found by Napier's Analogies, or from formulæ or rules derived from them. For this purpose let c be the complement of the latitude, and p the polar distance.

$$1.\ \tan \tfrac{1}{2}(m+e) = \cot \tfrac{1}{2} t \cos \tfrac{1}{2}(c \backsim p) \sec \tfrac{1}{2}(c+p),$$
$$2.\ \tan \tfrac{1}{2}(m-e) = \cot \tfrac{1}{2} t \sin \tfrac{1}{2}(c \backsim p) \operatorname{cosec} \tfrac{1}{2}(c+p).$$

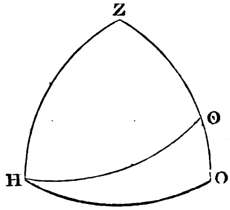
Hence $\frac{1}{2}(m+e) - \frac{1}{2}(m-e) = e$, the azimuth of the pole-star from the meridian referred to the horizon. Or let d be the declination of Polaris, and l the latitude of the place of observation,

$$3.\ \tan \tfrac{1}{2}(m+e) = \cot \tfrac{1}{2} t \cos \tfrac{1}{2}(d \backsim l) \operatorname{cosec} \tfrac{1}{2}(d+l),$$
$$4.\ \tan \tfrac{1}{2}(m-e) = \cot \tfrac{1}{2} t \sin \tfrac{1}{2}(d \backsim l) \sec \tfrac{1}{2}(d+l).$$

Since d and l remain constant during a series of observations made in one day, while t varies, the logs of the two last factors are constant, and this renders the computation of an azimuth by the pole-star remarkably easy.

18. In many cases of nautical surveying, the true bearing of any well-defined object at a considerable distance, and on, or nearly on, the same level with the eye of the observer, is required to be determined with a reflecting instrument. To perform this operation, bring the image of the sun to the object, and make its nearest limb accurately to touch the object, while at the same time with another instrument let the sun's altitude be taken. Correct the observed distance

for index-error, if necessary, and add the sun's semidiameter; the result will be the apparent distance of the sun's centre from the object. In like manner correct the sun's altitude for index-error, dip, and semidiameter, the result will be the sun's apparent altitude. Now, to compute the azimuthal angle between the sun and the object, there will be formed, when the object is on the same level with the eye, a quadrantal spherical triangle H Z ☉, of which the sides are the zenith distance ZH=90°, the sun's zenith distance Z☉, and the oblique observed distance H☉; to find the angle Z at the zenith, which is the difference between the bearings of the object and the sun.



Compute the sun's true azimuth from the altitude in the usual manner, take the sum or difference of these, according to circumstances, as indicated by their relative positions with respect to the meridian, and the true bearing of the object will be determined. If the bearing of the same object be taken with the azimuth compass, the variation of the compass will likewise be obtained. To determine the true bearing in this manner, it must be remarked that the sun's vertical motion should be as great as possible, or his position ought to be near the prime vertical, and that the object to which the sun is referred should be about 90° from the point of the horizon to which the sun is vertical. When this is impossible, the object should be chosen so that the angle which the observed arc or distance makes with the horizon, may not by estimation exceed 45°. When the object is elevated above the level of the eye, it is necessary to observe its altitude, and compute the angle at the zenith from the three sides of an oblique angled spherical triangle formed by the observed distance, and the zenith distances of the sun and the object whose azimuth is required; and this is in fact the first part of the method of reducing, by spherical trigonometry, the apparent distance to the true in lunar observations, that is, from the two apparent altitudes and apparent distance to find the angle at the zenith.

The azimuth of a point or signal, by means of the sun or a star, may be found readily when the time is accurately known. In this case there are given the polar distance and the hour angle, or that contained at the pole, to determine the angle at the zenith by the Analogies of Napier.

If the sun be the object, the angle at the pole is the complement of the time to 12^h in the forenoon, but the apparent time itself, if in the afternoon. If a star be observed, the angle at the pole is equal to the sidereal time *minus* the right ascension of the star, or equal to the apparent time *plus* the right ascension of the sun, *minus* the right ascension of the star. The polar angle is *minus* when the star is east of the meridian, *plus* when west.

When extreme accuracy is required in determining the azimuth of a signal, the observations of the angular distance, by the reflecting circle or by Borda's repeating circle, between the star and the signal should be made at the same instant with the zenith-distance of the star and the signal, if convenient, though that of the latter may be made at any time either preceding or following the observations, since, with the exception of refraction, it remains stationary. These distances are the apparent distances as affected by parallax and refraction. If the zenith-distance of the star cannot be conveniently observed at the same time when the angular distance between the star and signal are taken, it may be calculated by spherical trigonometry, as will be afterwards shewn, taking care to apply the effects of refraction and parallax to find the apparent zenith-distance with a contrary sign to that used in finding the true.

When the azimuth is found by observations on the pole-star, or similar methods, the horizontal circle must be read at the same time with the vertical, in order to compare the azimuth of the star with a referring lamp, and from this, at any convenient opportunity, other conspicuous points selected as stations in the general survey.

19. I shall now proceed to illustrate these rules and formulæ by practical applications. Having determined the error and rate of my chronometer, as previously exemplified, the following observations were made at Inchkeith Lighthouse, to determine the latitude and direction of the meridian by the pole-star. For this purpose I resided on the island a few days, during which I made several observations on the heights in the vicinity of Edinburgh, as well as some on the latitude by the sun and α Aquilæ, for which a special table was drawn up in the manner already explained, by which the reduction to the meridian for the distance t was made by inspection. I chiefly trusted those made on the 21st of August upon the pole-star, which I continued to observe from about 10 o'clock in the evening to 1 o'clock next morning. During this period I completed eight series of double observations, reversing the circle each time, or sixteen single observations, comprehending forty-eight readings of the verniers on each circle, accompanied by the times of observations, and the readings of the level. The circles used were six inches in diameter, having each three verniers reading to 10″, and a level whose divisions each indicate 2″. Having made these preliminary remarks, so that every thing relative to my operations may be fully understood, I shall record the first series of observations, and perform the computations at full length, so as

to render the whole operation clear and distinct to every one having a very ordinary knowledge of such subjects. In this record *b* signifies the height of the English barometer, *τ* the temperature by its attached or interior thermometer, *t* the temperature of the air by the exterior thermometer, in degrees of Fahrenheit; Ver., the different verniers of the respective circles marked A, B, C; Z. D. the observed zenith-distance; H. D. the horizontal angular distance to the referring lamp; I. M. T. Inchkeith mean time; G. M. T. Greenwich mean time; S. T. G. M. N. sidereal time at Greenwich mean noon, &c.

POLARIS.

Inchkeith, August 21. 1840, $b=29^{in}.70$, $\tau=64°$, $t=64°$ Error of Chronometer at $10\frac{1}{4}^h$ P.M. fast 1^m $58^s.4$, rate $19^s.7$ gaining,

Obs.	Times.			Ver.	Z. D.			Ver.	H. D.			Level. +	—
	h.	m.	s.		°	′	″		°	′	″	e	o
1.	10	9	5	A	33	34	40	A	61	54	55	24	22
				B		34	25	B		54	50		
				C		34	30	C		54	55		
2.	10	19	35	A	33	27	20	A		54	60	22	23
				B		27	20	B		54	45		
				C		27	30	C		54	55		
Means	10	14	20		33	30	55.8		61	54	53.3	46	45
Error Cr.	—	1	58.4	l =	+		0.5					45	
I. M. T.	10	12	21.6		33	30	56.3					2) 1	
Long. I.	+	12	32.0	r =	+		37.2						
G.M.T.	10	24	53.6	T.Z.D.	33	31	33.5 = δ					+0″.5 = l	

Refraction.

Z. D.	33° 31′	log $\delta\,\theta$.	1.5876
b =	29.70	log .	9.9956
τ =	64°	log .	9.9994
t =	64	log .	9.9875
r =	37″.2 .	log	1.5701

Sidereal Time, t

	h.	m.	s.
S. T. G. M. N.	9	59	29.28
I. M. T. . . .	10	12	21.60
Red. to G.M.T. +		1	42.65
Sid. time obs.	20	13	33.53
Star's R. A. .	1	2	36.78
t . . . . =	19	10	56.75

p	1° 32′ 33″.8	tan	8.4302701	sec	0.0001575	tan	8.4302701
t	$19^h 10^m 56^s.75$	cos	9.4837861	. . .		sin	9.9788502
$u-$	0° 28′ 12″.28	tan	7.9140562	cos	9.9999854		
			δ=33° 31′ 33″.5	cos	9.9209762		
λ	56 30 9 .00	. . .		sin	9.9211191	sec	0.2581391
l	56 1 56 .72	m	=	N. 2° 39′ 40″.15 E		tan	8.6672594

In the same manner the remaining parts of the series were computed by the formulæ in § 16.

But since the star moves in a circle, the mean zenith-distance and horizontal angle is not that at the middle of the arc described during the interval between the observations, as it ought to be, and, by investigation, the following corrections must be applied to the latitude and azimuth.

$$dl = (p'' \sin 1'' \cos t + p''^2 \sin^2 1'' \cos 2t \cot \delta) f \quad . \quad . \quad (11)$$

$$= p'' \sin 1'' \cos t f; \text{ in this case,}$$

$$dm = -p'' \sin 1'' \sin t \sec l f \quad . \quad . \quad . \quad . \quad . \quad (12)$$

in which f is the factor, from Table XVII.

p''=1° 32′ 33″.8=5553″.8	log	. . . .	3.744590
sin 1″ . . .	log	. . . .	4.685575
p'' sin 1″ . .	log	. . . .	8.430165
l=56° 1′ 59″.6 .	log secant	. . .	0.252812
p'' sin 1″ sec l . .	log	. . . .	8.682977

	h. m. s.
1st Observation	10 9 5
2d Observation	10 19 35
Mean . . .	10 14 . 20

Mean − 1st=	5 15 V=2623 log	. .	3.418798
Log V′ for one observation		. . . . .	8.314425
Log f . .	1.733223	. . . . .	1.733223
$t=19^h\ 10^m\ 56^s.75$ cos	9.483786	sin t	9.978850
p'' sin 1″ log .	8.430165	p'' sin 1″ sec l, log .	8.682977
dl= +0″.44 log .	9.657174	dm= +2″.48 log .	0.395050

In this way the corrections were computed for the whole series, and the final results are as follow :—

No.	l′ ° ′ ″	dl′ ″	l ° ′ ″		Successive values. ° ′ ″
1	56 1 56.72	+ 0.44 =	56 1 57.16	. .	56 1 57.16 N
2	2 1.14	+ 0.33 =	2 1.47	. .	1 59.31
3	2 2.04	+ 0.69 =	2 2.73	. .	2 0.44
4	2 2.00	+ 0.74 =	2 2.74	. .	2 1.03
5	1 56.95	+ 0.30 =	1 57.25	. .	2 0.26
6	1 58.30	+ 0.21 =	1 58.51	. .	1 59.98
7	1 59.70	+ 0.29 =	1 59.99	. .	2 0.12
8	2 0.10	+ 0.49 =	2 0.59	. .	2 0.06
	Reduction to centre of tower for 25 feet				— 0.24
	True latitude			. . . .	56 1 59.82 N

Azimuth of Light.

No.	H. D. ° ′ ″		m′ ° ′ ″	dm ″	m ° ′ ″
1	61 54 53.3	+	2 39 40.15	+ 2.48 =	N 64 34 35.93 E
2	62 0 40.5	+	2 33 49.35	+ 1.24 =	31.09
3	62 8 3.7	+	2 26 35.20	+ 2.16 =	41.06
4	62 14 58.0	+	2 19 31.10	+ 1.92 =	31.02
5	62 21 15.0	+	2 13 24.80	+ 0.69 =	40.49
6	62 27 7.4	+	2 7 27.60	+ 0.43 =	35.43
7	62 33 0.0	+	2 1 37.20	+ 0.52 =	37.72
8	62 40 14.5	+	1 54 17.20	+ 0.66 =	32.36

Mean azimuth	N 64 34 35.64 E
Reduction to centre of tower . .	— 35.70
Azimuth at centre of tower . .	N 64 33 59.94 E
Angle to Observatory	134 9 7.30
Observatory bears from lighthouse .	198 43 7.24
	180 0 0.00
Observatory bears from Inchkeith .	S 18 43 7.24 W
Convergence of the meridians c'' .	— 2 21.50
Inchkeith bears from Observatory .	N 18 40 45.74 E

The method of computing c'' will be subsequently given.

The Trigonometrical Survey station is S. 27° 27′ W., distant 6.855 feet from the centre of the dome or pillar, therefore,

		°	′	″	
To the bearing of Inchkeith above . .	N	18	40	45.74	E
Add reduction		+		6.74	
Inchkeith bears from Trigonomet. Survey station	N	18	40	52.48	E
By Trigonometrical Survey . . .		18	40	53.50	
Mean		18	40	52.99	

20. Having now shewn the method of preparing the observed horizontal angles for computation, of fixing the latitude of any selected point, and the bearing of another from it, I shall now give a few rules and formulæ for deducing from these, and an extended triangulation, the latitude, longitude, and azimuth of the principal points of the series,—reserving the computation of heights to a succeeding part of this essay.

In deducing latitudes, longitudes, azimuths, and heights geodetically, it is necessary to be enabled to convert readily any distance measured in feet on the earth's surface into arcs; and hence the radius of curvature of the measured arc, in any given position on the terrestrial spheroid, is required by the principles of the conic sections.

Now, the radius of curvature is to an arc R″, equal to the radius in seconds, as the distance in the same measure with the radius of curvature is to the corresponding arc in seconds. Let A″ be the required arc in seconds, corresponding to A, any measured arc on the earth's surface in feet, to which r is the radius of curvature,

$$r : A :: R'' : A'' \text{ or } A'' = \frac{R''}{r} \times A \quad . \quad . \quad . \quad (13)$$

Wherefore, if M be the factor to convert a curvilineal distance on the meridian into seconds of arc; P that on the perpendicular to it; and O that on any oblique arc, making an angle α with the meridian; then, if a denote the radius of the equator, b the polar semiaxis, e the eccentricity, and l the latitude ;

$$M = \frac{R''}{a^2 b^2} (a^2 \cos^2 l + b^2 \sin^2 l)^{\frac{3}{2}} = \frac{R''}{a(1-e^2)} (1 - e^2 \sin^2 l)^{\frac{3}{2}} \quad . \quad . \quad (14)$$

$$P = \frac{R''}{a^2} (a^2 \cos^2 l + b^2 \sin^2 l)^{\frac{1}{2}} = \frac{R''}{a} (1 - e^2 \sin^2 l)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$O = M \cos^2 \alpha + P \sin^2 \alpha = P \frac{1 - e^2 (1 - \cos^2 l \cos^2 \alpha)}{1 - e^2} \quad . \quad . \quad . \quad (16.)$$

From these formulæ Tables XIX., XX., and XXI. have been computed, the coefficient for terrestrial refraction n, in the two last, having been taken equal to 0.08 or $\frac{1}{12.5}$ of the intercepted arc, which is a sufficient approximation to the truth in ordinary atmospheric circumstances.

21. Previously to the determination of heights trigonometrically, the curvilineal distance, or its chord at the level of the sea, ought to be augmented for the height of the lower station, since the radii from the centre through their summits diverge proportionally to that height. This correction may be obtained from the following formula, or the results derived from it arranged in a table. Let K be the chord of the augmented arc A at the height h, derived from the arc a at the level of the sea, then

$$\text{Log } K = \log a + \frac{M h}{\rho} - \frac{M a^2}{24 \rho^2} = \log a + m h - p a^2 \quad . \quad . \quad (17.)$$

From this formula Table XXIV. was computed. The number S is the difference of the log secant of half the angle v between the verticals and log $p a^2$, which contributes to greater accuracy in considerable heights. I shall now give the necessary formulæ and rules to find latitudes, longitudes, and heights geodetically.

Explanation of Symbols, with their Values.

A = the measured arc in feet on the surface of the Terrestrial Spheroid.
R″ = an arc equal to the radius in seconds, . log 5.3144251
a = the radius of the equator in feet, . . log 7.3206165
b = the polar semiaxis in feet, . . . log 7.3191664

$$e = \left(\frac{a^2 - b^2}{a^2}\right)^{\frac{1}{2}} = \frac{\{(a+b)(a-b)\}^{\frac{1}{2}}}{a} = 0.0815815, \quad \log \bar{2}.9115918$$

$\epsilon = \frac{1}{2} e^2 + \frac{1}{8} e^4 + \&c. = \frac{1}{300}$ = elliptically, . log $\bar{3}$.5228787
$f = a\,e = 1706900$ feet . . . . log 6.2322083
$c = a - b = a\,\epsilon = 69742$ feet, . . . log 4.8434944
l = the given latitude farthest from the equator.
l' = the required latitude nearest the equator.
λ = the latitude of the foot of the perpendicular from the required point, upon the meridian passing through the given point.
z = the given azimuth.
z' = the required azimuth.
$\Delta\, l$ = the difference of latitude.
Δp = the difference of longitude.
Δz = the difference of azimuth or convergence of the meridians passing through the given and required points.

Making $\frac{A R''}{a} = a''$, we shall have, from an investigation that cannot be conveniently given here,

(1.) $\Delta\, l = -a''(1 + 2\,\epsilon - 3\,\epsilon \sin^2 l) \cos z + a''^2 \frac{1}{2} \sin 1'' \tan l \sin^2 z$ (18)
(2.) $\Delta p = a''(1 - \epsilon \sin^2 l) \sin z \sec l - a''^2 \sin 1'' \sin z \cos z \tan l \sec l$ (19)
(3.) $\Delta z = a''(1 - \epsilon \sin^2 l) \sin z \tan l' + a^2 \frac{1}{2} \sin 1'' \sin z \cos z$ . . (20)

These are the principal formulæ generally required. In addition to these, that for determining an oblique arc may be added,

4. $\Delta O = a''(1 - \epsilon \sin^2 l + 2\,\epsilon \cos^2 l \cos^2 \alpha)$ . . . . . . (21)

Log sin 1″ = 4.685575, log ½ sin 1″ = 4.384545

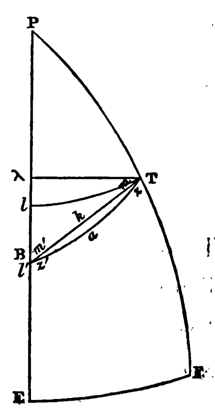
Introducing the values of M, P, and O, of which the logarithms are given in Tables XIX., XX., and XXI., according to the directions given along with them, making first r'' equal to the reduction of λ to l, derived from the last part of formula (18), given in Tables XXII. and XXIII.

$$(4.)\ \Delta l = -\text{AM} \cos z + r'' = \text{AM} \cos m - r'' \quad . \quad . \quad . \quad . \quad . \quad (22)$$
$$(5.)\ \Delta p = \quad \text{AP} \sin z \sec l' = \text{AP} \sin m \sec l' \quad . \quad . \quad . \quad . \quad . \quad (23)$$
$$(6.)\ \Delta z = \quad \Delta p \sin \tfrac{1}{2}(l + l') \sec \tfrac{1}{2}(l - l') \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

In north latitudes, the azimuth z is generally reckoned from the south towards the west or east, and is the supplement of m, or that reckoned from the north, in the application of which attention must be paid to the signs. Indeed, in some operations, the azimuth is reckoned from the east westwards round the whole circle, in accordance with which the arguments to Tables XIX., XX., and XXI. have been so given.

PRACTICAL RULES.

22. To illustrate the method of employing these formulæ and tables in calculation, let P be the north pole in this instance, E a point in the equator, B a point of which the latitude and longitude are known, T another place whose bearing and distance from B are given, and from these the latitude and longitude of T and the azimuth of B from T, are required. Also, let PBE be the meridian passing through B, PTF the meridian passing through T, PBT the azimuth denoted by α in the formulæ, or m' or z' in the tables, BT the distance or curvilineal arc a in feet, of which the chord is k, Tλ a



perpendicular from T, the required point upon the meridian passing through the given point B, the distance from the foot of which to the equator, measured by Eλ, is the latitude of λ; l' the latitude of the place nearest the equator, l that of the more distant, and Tl the parallel of latitude passing through T, making E l the latitude of T, or that required. The very small arc $\lambda\, l$, called the reduction of λ to l in Tables XXII. and XXIII., must always be subtracted from λ to give l'.

If this small arc exceeds the limits of the tables, it may be computed. For this purpose, it may be observed that if p'' be the perpendicular arc, then $p''=\mathrm{AP}\sin z$, the argument to find r'' from the tables. But, independent of the tables,

$$r''=\mathrm{A}^2\mathrm{P}^2\sin^2 z\,\tfrac{1}{2}\sin 1''\tan l=p''^2\,\tfrac{1}{2}\sin 1''\tan l \quad . \quad . \quad . \quad (25)$$

the formula from which the tables were constructed, and may supply their place in cases beyond their limits.

It must likewise be observed that Bλ is a small arc of the meridian to be added to the given latitude in proceeding towards the pole, or subtracted when receding from it, to give the latitude of the foot of the perpendicular λ, the argument for taking the log P from the tables. The argument to obtain log M is half the sum of the latitudes approximately, or $\frac{1}{2}(l+l')$, to be derived from a provisory calculation, in order to get the mean latitude between the given stations. The number of *minutes* to be added to the smaller latitude l', or subtracted from the greater l, to get $\frac{1}{2}(l+l')$, may be computed by the following rule.

To the constant log 5.914630, add the log of the meridian-distance in feet, the sum will be the log of half the difference of latitude in minutes, or $\frac{1}{2}(l-l')$, to be added to l', or subtracted from l, to give $\frac{1}{2}(l+l')$, the middle latitude sufficiently near the truth for taking log M from the tables.

1. By a provisory calculation, such as that just given, or by a repetition of the more accurate method now to be shewn, if thought necessary, find the middle latitude, or $\frac{1}{2}(l+l')$.

2. To the logarithm of the curvilineal distance, or arc a, add the log cosine of the azimuth, or m, and the log M from

Table XIX., answering to the mean latitude, or $\frac{1}{2}(l+l')$, the sum will be the logarithm of an arc of the meridian in seconds m'', to be added to the latitude l' if approaching the pole, but subtracted from l if receding from it, the sum or difference will give λ, the latitude of the foot of the perpendicular upon the given meridian from the point in that required.

3. To the log of a add the log sine m, the azimuth, the log P answering to λ, the sum will be the log p'', the perpendicular arc in seconds.

4. To the constant log 4.384545 (the log $\frac{1}{2}\sin 1''$) add log tan λ and twice the log p'', the sum will be log r'', the reduction of λ to l *always subtractive.* This may be also taken from Tables XXII. or XXIII., if within the limits of the tables. It may be observed, that four times r'', answering to $\frac{1}{2}p''$ will be the reduction to p'' nearly, which will extend the table, and the results will not differ much from the truth. This, at least, will be a check to calculation.

5. To the log tangent p'' add the log secant l', the sum will be the log tangent Δp, the difference of longitude, which, properly applied to the longitude of the place of observation, will give the longitude of the point required.

6. To log tangent Δp add log sine $\frac{1}{2}(l+l')$ and the log secant $\frac{1}{2}(l-l')$, the sum will be the log tangent Δz, the convergence of the meridians of the given and required points, which, added to the azimuth m', at the latitude nearest the equator, will give m, or rather z, the azimuth at the latitude farthest from it, and *vice versa.*

7. To the log O, answering to the middle latitude and given azimuth α, from Table XIX., add the log of the given distance a, the sum will be the log of the intercepted arc in seconds, which measures the angle between the verticals of

the given points. If the log O be taken from Table XX., the result will be angles of the verticals diminished by the effect of refraction, taken at 0.08, of the intercepted arc. The log O from Table XXI. is the log of $\frac{1}{2}\rho(1+n)^2$, employed in the computation of heights by the depression of the horizon of the sea, the mean value of n being 0.08 as before. By these rules the position of any number of points may be fixed; but in practice a different arrangement is frequently followed.

Suppose a parallel to the meridian of Edinburgh and to its perpendicular to be drawn through each station, we have the bearings and distances of the other stations from such parallels, calculated by means of a right-angled plane triangle, of which the distance or hypotenuse, and the bearings or one angle, are given, to find the other two sides. Thus, let k be the distance, m the azimuth, and ϵ the spherical excess, we have strictly a triangle deviating slightly from a right-angled triangle, when the spherical excess is applied, but in all ordinary cases of practice the latter may be safely omitted. Now, if x be the distance from the parallel to the perpendicular on the meridian in feet, y the distance from the parallel to the meridian also in feet, introducing ϵ, we have

1. $x=k\cos\left(m-\frac{2}{3}\epsilon\right)$. 2. $y=k\sin\left(m-\frac{1}{3}\epsilon\right)$

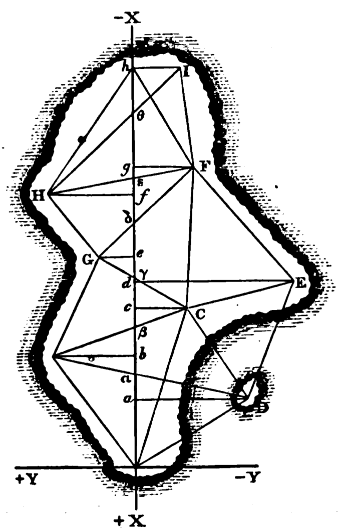
Omitting ϵ, as is the general practice, and

3. $x=k\cos m$. 4. $y=k\sin m$.

This may be permitted, because each determination of a point is an independent operation, and is not affected by an accumulation of errors.

23. I shall now give a general outline of the method of conducting the survey of a country or of an island on the preceding principles. In this case, it is necessary to determine

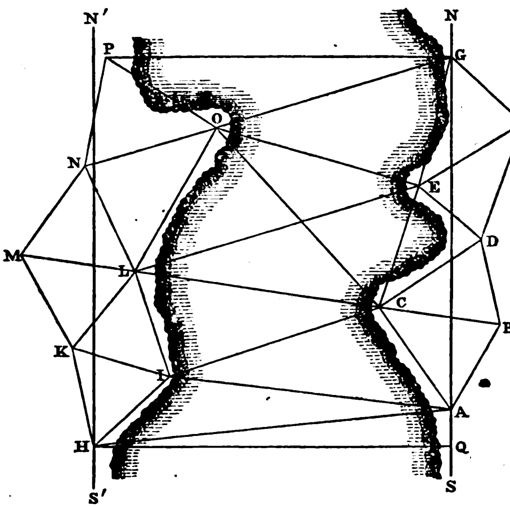
the latitude, longitude, and direction of the meridian of any convenient point A, as has already been shewn, with reference to a side of one or more of the triangles, such as *b* AB, or *c* AC, &c. It will then be necessary to throw a series of judiciously chosen triangles over the surface of the island and adjacent islets as may be near its coasts, such as ABC, CBG, &c., so as to embrace the chief features of the whole island. These points must next be referred to the principal meridian by means of perpendiculars let fall from each point upon it, thus forming the abscissæ +X, −X, &c. to the



south and north of the point A, and ordinates parallel to the perpendicular to it +Y, −Y, &c. to the west and east of the same meridian. These are represented by A*a*, A*b*, &c. and D*a*, B*b*, &c. by drawing temporary parallels to +X, −X, &c. +Y, −Y, &c. throughout the whole compass of the survey; those abscissæ to the south of A being conditionally reckoned positive, those to the north negative; while those ordinates to the east of A are considered negative, and those to the west positive. If a distance as AB cannot be deduced from an adjacent survey with sufficient precision, then a fundamental base in some convenient situation must be measured with great care, and connected with some of the sides trigonometrically, from which the sides of the whole series of triangles must be deduced by calculation, as formerly shewn. This is, for the sake of distinction, called the *primary* triangulation, in which the sides of the triangles extend from about 30 to 50, or even occasionally to 100 miles. These larger triangles are next broken down into a smaller class, called the *secondary*, whose sides are limited to about 10 or 15 miles, in which the angles may be measured with somewhat inferior instruments. The intermediate points are then filled in by the five-inch theodolite, the surveying compass, and the chain, which may be called the tertiary triangulation, and concluding process.

24. In a similar manner may a survey of the adjacent coasts of a strait, firth, or river, be completed, and the bearings and distances of corresponding points on opposite sides be laid down, whether they be visible from each other or not. This may be readily done in various ways, one of which is, to run two parallels or two meridians of known distance from each other, as may be most convenient under given circumstances, and, by finding the position of each station on its own meridian, that is, its distance on the meridian from a given point in it, and the perpendicular from it upon that meridian, then these will afford the means, by the solution of a triangle, to find the bearings and distances of all or any of them, in such directions as it may be thought necessary or convenient to lay down soundings, leading marks, dangers, &c.; by which means the nautical surveyor will be enabled to complete his chart in a satisfactory manner.

Let NS, N′S′ be the two conventionally chosen meridians by one or more surveyors, whose operations embrace the opposite shores of a river or strait, where it is possible and safe to have the necessary piles and staffs erected on shore, then the perpendicular distance HQ being found by



observations taken on purpose, the points A, C, E, G; H, I, L, O, P, may be referred to their respective meridians NS, N′S′, as in the preceding figure. By the solution of the right-angled plane triangle HQA, right-angled at Q, having AQ, QH given, the angles QAH, AHQ may be found, together with the side AH. Hence the angle IHA may be found, consequently with the sides IH, HA, and the contained angle IHA, the side IA may be found whether the point A be visible from H and I or not. Now the angle CAQ being known, and IAQ having been found by computation, the angle CAI becomes known. Whence, with the given sides IA, AC, and the contained angle IAC, the side IC may be determined.

It is clear that this method, combined with others easily deduced, may be followed through the whole series; from which the form and contour of the shores and distances on which soundings, dangers, &c. should be placed or laid down on a chart are readily inferred. Should the survey be carried on in a foreign country, or barbarous shores, where, from danger, the necessary marks cannot be safely erected on shore, the masts of lighters, boats, and barges, properly secured, may be used as signals, especially if they have polished frusta of cones of zinc-plates, or sheets of block-tin fixed to the mast-head. These may have the greater diameter about nine inches, the less six, and the height twelve, or in these proportions nearly, greater or less according to the distance. These will reflect the sun's image readily to the observer, even in thick weather, whence the angles will be obtained in an easy and satisfactory manner, when the observer and the objects are in a proper position, the time of which must be estimated and carefully watched.

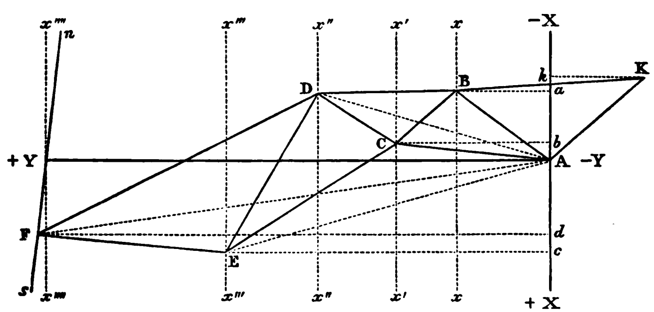
If the lines of reference assumed are parallels of latitude, they will continue equidistant, but if meridians, they will converge towards the pole, and diverge towards the equator, and the distance between them will vary as the radius of the parallel. In nice operations of considerable extent, this variation cannot be neglected, though in those of smaller magnitude, it will be so inconsiderable, as, in ordinary circumstances, to be of little consequence. To take this into account when thought necessary, let R be the radius of curvature of any parallel whose latitude is l,

$$R = \frac{a \cos l}{(1 - e^2 \sin^2 l)^{\frac{1}{2}}} = a \cos l \, (1 - \varepsilon \sin^2 l) \quad . \quad . \quad . \quad (26)$$

Whence, by computing R for the parallels of l and l', the distance at l may be reduced to that at l'.

25. If, however, a parallel to the primary meridian be assumed, the operation will be more simple, as it will be unnecessary to compute the convergence in feet, or their distance at different latitudes, while the latitudes, longitudes, and azimuths may still be readily found by the preceding rules, and this is the method generally adopted.

Let XX be the meridian passing through the Observatory of Edinburgh A, YY a perpendicular to it, $x\,x$, $x'\,x'$, &c. parallels to XX, passing through the stations, B Bencleugh, C Bencampsie, D Benlomond, E Goatfell, E Cairn Aird in Islay, and K Kellylaw in Fife. Hence there are formed the triangles ABC, BCD, DCE, and EFD, which are treated as already directed, pages 5, 6, &c. Having determined the bearing of Bencleugh, or angle BAX$=\alpha$, and the distance AB$=k$, there may be found A$a=-x$ and B$a=+y$, and so on to the last triangle AFd; from which the absciss A$d=+x$, and F$d=+y$ are obtained from a combination of all the intermediate triangles computed in a similar manner, and the results are stated in a table.



Though the signs in the following examples are those employed by many engineers, especially on the Continent, yet in my opinion it would probably be better to make those of *x positive* when they increase the latitude, and negative when they diminish it. The lines $x\,x$, &c. being all parallel to XX, are therefore, it must be recollected, not meridians. The latter meet at the poles, and consequently are inclined to one another at certain angles. Thus the meridian $n\,s$ is inclined to XX at an angle $n\mathrm{F}x''''$ of 2° 34′ 17″, which is called the convergence of the meridians, Δz, and varies with the latitudes and difference of longitude as computed by formula (6), page 37, and recorded for each station in the table, if thought necessary. It must be properly applied to the bearings, such as BAa, so as to get the bearing of A from B, called technically, especially by marine surveyors, *the back bearing.*

Having established these general principles, we shall now illustrate the whole by practical examples. By our operations at Inchkeith, in latitude 56° 1′ 59″.82 N., longitude 12^{m} 32^{s} W., we have found that Edinburgh Observatory bears S 18° 43′ 7″.24 W., distant 30272 feet, it is therefore required to find the latitude and longitude of Edinburgh Observatory, and thence the position of the Trigonometrical Survey Station, in order to connect these triangles with it, so that the results in our examples may be comparable with those in the survey. It must be observed that Cairn Aird was not observed from Goatfell, and that I merely computed the bearing and distance from the approximate latitudes and longitudes by Ivory's formula, as given in my Mathematical and Astronomical Tables. Consequently these results are to be considered as approximative only.*

* From these circumstances, though the position is given in the figure, the numerical results are not stated in the following table.

	°	′	″	
Inchkeith Light, latitude, l . . .	56	2	0	N
Edinburgh Observatory, latitude, l' . .	55	57	16	N
Sum, or $l+l'$	111	59	16	
Half, or $\frac{1}{2}(l+l')$	55	59	38	
Difference, or $l-l'$		4	44	
Half, or $\frac{1}{2}(l-l')$		2	22	
Edinburgh Observatory, longitude . .	3	10	46	W
Inchkeith Light, longitude	3	7	56	W
Difference, or Δp		2	50	

These being points pretty well known, their latitudes and longitudes will therefore turn out, by a geodetical computation, the same nearly as stated above.

This operation is performed by the formulæ given at page 37, or the subsequent practical rules, by the aid of the Tables XIX., XX., XXI., XXII., and XXIII.; for the method of using which tables, their explanation must be consulted.

$\frac{1}{2}(l+l') = 55°\ 59'\ 38''$ log M = 7.9937224 λ gives log P = 7.9928141
$z = 18\ 43\ 7.24$ cos 9.9763985 sine . 9.5063992
$A = 30272$ feet log 4.4810411 . . 4.4810411

$m'' = -0°\ 4'\ 42''.59$ log 2.4511620 $p'' = 1'\ 35''.6$ log 1.9802544
$l = 56\ 1\ 59.82$

$\lambda = 55\ 57\ 17.23$
$r'' = -\ 0.03$ *

$l' = 55\ 57\ 17.20$ secant 0.2519306

$\Delta p = 0\ 2\ 50.68$ log 2.2321850
$L' = 3\ 7\ 56.00$ $\frac{1}{2}(l+l') = 55°\ 59'\ 38''$ sine 9.9185430

$L = 3\ 10\ 46.68$ $\Delta z = 0°\ 2'\ 21''.49$ log 2.1507280
$z = 18\ 43\ 7.24$

$m' = $ N $18\ 40\ 45.75$ E.

* This correction, r'', may be readily taken from Table XXIII. in general. It may also be computed by the formula (25), page 38.

Log p'' and 2 = ...	3.96051
Log $\frac{1}{2}$ sin 1″	4.38455
Tan λ	0.17028
$r'' = 0''.033$	8.51534

which is always small, when the difference of longitude is not great.

Hence l', the latitude of Edinburgh Observatory, is 55° 57′ 17″.20 N., longitude L = 3° 10′ 46″.68 W., and the bearing of Inchkeith light from Edinburgh Observatory, or m', is N. 18° 40′ 45″.75 E., agreeing with the result in pages 33, 34. Hence, as is stated there, from the Trigonometrical Survey Station, near the pillar in the Observatory,

		°	′	″	
Inchkeith Light bears . . .	N.	18	40	53.00	E.
Angle, Inchkeith, Calton, Bencleugh,		73	16	29.28	
Bencleugh bears from Calton Station	N.	54	35	36.28	W.
Angle, Bencleugh, Calton, Bencampsie,		28	43	26.47	
Bencampsie bears from Calton Station	N.	83	79	2.75	W.

		°	′	″	
Bencleugh bears from Calton . .	N.	54	35	36.28	W.
Benlomond, Calton, Bencleugh, .		18	36	57.00	
Benlomond bears from Calton . .	N.	73	12	33.28	W.
Also, by observation, Kellie Law bears from Calton Station . . .	N.	37	23	4.00	E.

With the distances in feet from the Calton to these different points, their latitudes and longitudes may be found in the manner just shewn.

From the Trigonometrical Survey Station on the Calton Hill, then, there will be obtained

		°	′	″		
1. Kellie Law bears	N.	37	23	4.00	E.	distant 135083.5 feet.
2. Bencleugh bears	N.	54	35	36.28	W.	distant 146334.6 feet.
3. Bencampsie bears	N.	83	19	2.75	W.	distant 196909.0 feet.
4. Benlomond bears	N.	73	12	33.28	W.	distant 308307.6 feet.

Though the preceding *data* are sufficient to fix the positions of the respective points recorded, yet we shall treat the whole in a systematic manner as a small arc of a parallel across the country, in order to exemplify the method of conducting such operations, and deducing the results successively from each other.

Commencing at the Trigonometrical Survey Station on the Calton Hill, fixed by our previous deductions, we shall now determine the positions of the places recorded above, beginning with that of Kellie Law.

1. Constant logarithm, p. 38		. . . . .	5.914630
α=N. 37° 23′ E. cosine		. . . . .	9.900144
A=135083.5 feet		. . . . . .	5.130602
$\frac{1}{2}(l-l')$	+ 0° 8′.8 N.	m' log	0.945376
l'	+ 55 57.3 N.		
$\frac{1}{2}(l+l')$	56 6.1 N.		

This preliminary step is only an approximation to the middle latitude, or $\frac{1}{2}(l+l')$, in order to get the argument to take the logarithm of the factor M from Table XIX. for converting feet on the surface of the earth into seconds of arc, to determine l accurately when l' is known.

$\frac{1}{2}(l+l')$=	56° 6′ 6″	log M=	7.9937149	λ gives log P,	7.9928071
m=	37 23 4	cos .	9.9001374	sine . .	9.7833033
A=	135083.5	log .	5.1306020	. . .	5.1306020
m''=	+0° 17′ 37″.92	log .	3.0244543	p''=13′ 26″.7 log	2.9067124
l'=	55 57 17.20				
λ=	56 14 55.12				
r''=	− 2.36 *				
l=	56 14 52.76	secant			0.2552382
Δp=	−0° 24′ 11″.95 E.	log			3.1619506
L=	3 10 46.68 W.	$\frac{1}{2}(l+l')$=56° 6′ 4″.98		sin	9.9190916
L′=	2 46 34.73 W.	Δz= + 0° 20′ 5″.15		log	3.0810422
		m=N. 37 23 4.00 E.			

Bearing of Calton, or z= S. 37 43 9.15 W. from Kellie Law.

* Log $p''\times 2$=	—5.81342
Log $\frac{1}{2}$ sin 1″=	4.38455
λ=56° 15′ tan	0.17511
r''= −2″.36 log	0.37308

2. Constant logarithm			5.91463
$\alpha = 54^\circ\ 36'$ cosine			9.76289
$A = 146334.6$ log			5.16535
$\frac{1}{2}(l-l') + 0^\circ\ 7'.0$			log 0.84287
l' 55 57.3			
$\frac{1}{2}(l+l') = 56\ \ 4.3$	log M = 7.9937170		log P 7.9928086
$m = 54\ 35\ 36.28$	cos 9.7629597		sine 9.9111902
$A = 146334.6$	log 5.1653471		5.1653471
$m'' = +0^\circ\ 13'\ 55''.65$ log	2.9220238	$p'' = 19'\ 33''$	log 3.0693459
$l' = 55\ 57\ 17.20$			
$\lambda = 56\ 11\ 12.85$			
$r'' = -\ \ 4.98$			
$l = 56\ 11\ \ 7.87$ secant			0.2545430
$\Delta p = +0^\circ\ 35'\ \ 8''.09$ log			3.3238889
$L' = 3\ 10\ 46.68$	$\frac{1}{2}(l+l') = 56^\circ\ 4'\ 12''.53$		sine 9.9189323
$L = 3\ 45\ 54.77$	$\Delta z = 0^\circ\ 29'\ \ 9''.13$		log 3.2428212
	$m = 54\ 35\ 36.28$		

Bearing of Calton, or $z =$ S. 55 4 45.41 E. from Bencleugh.

In the same manner may the computations of the positions of the other points be performed.

We shall, however, here determine the position of Bencampsie from Bencleugh, and then that of Benlomond from Bencampsie, whence, in a similar manner, may any number of points be fixed in succession.

Angle, Calton, Bencleugh, Bencampsie	$105^\circ\ 33'\ 43''.68$
Calton bears from Bencleugh	S. 55 4 45.41 E.
Bencampsie from Bencleugh bears distant 98240.3 feet.	S. 50 28 58.27 W.

3. Constant logarithm			5.91463
$\alpha = 50^\circ\ 29'$ cosine			9.80366
$A = 98240.3$ feet log			4.99229
$\frac{1}{2}(l-l') = -\ \ 5'.1$ S. log			0.71058
$l = 56\ \ 11.1$ N.			
$\frac{1}{2}(l+l') = 56\ \ 6.0$	log M 7.9937150	log P	7.9928126

$\frac{1}{2}(l+l')=56°\ 6'.0$	log M	7.9937150	log P	.	7.9928126
$Z=50°\ 28'\ 58''.27$	cos	9.8036681	sine	.	9.8872989
$A=98240.3$	log	4.9922897	log	.	4.9922897
$m''=-0°\ 10'\ 16''.13$	log	2.7896728	$p''=12'\ 25''.4$	l	2.8724012
$l=\ 56\ 11\ \ 7\ .87$					
$\lambda=\ 56\ \ 0\ 51\ .74$					
$r''=\ -\ \ \ \ 2\ .00$					
$l'=\ 56\ \ 0\ 49\ .74$	secant	.	.	.	0.2525937
$\Delta p''=+0\ 22\ 13\ .51$	log	.	.	.	3.1249949
$L'=\ 3\ 45\ 54\ .77$	$\frac{1}{2}(l+l')=56°\ 5'\ 58''8$			sine	9.9190828
$L=\ 4\ \ 8\ \ 8\ .28$	$\Delta z=-0°\ 18'\ 26''.82$			log	3.0440777
	$z'=\ 50\ 28\ 58\ .27$				

Bearing of Bencleugh, $Z=$ N. 50 10 31 .45 E. from Bencampsie.

	° ′ ″
Angle, Benlomond, Bencampsie, Bencleugh, by observation, is . . .	107 22 41.33
Bencleugh from Bencampsie bears . .	N. 50 10 31.45 E.
Benlomond from Bencampsie bears . .	N. 57 12 9.88 W.

4. Constant logarithm	5.91463
$\alpha=57°\ 12'$ cosine	9.73377
$A=119569$ feet log	5.07762
$\frac{1}{2}(l-l')=\ 0°\ \ 5'.3$	0.72602
$l'=56\ \ 0.8$	

$\frac{1}{2}(l+l')=56\ \ 6.1$	log M=	7.9937150	log P	.	=7.9928085
$m=57°\ 12'\ 9''.88$	cos	9.7337331	sine	.	9.9245855
$A=119569$ feet	log	5.0776186	log	.	5.0776186
$m''=+0°\ 10'\ 38''.36$	log	2.8050667	$p''=16'\ 28''.6$	l	2.9950126
$l'=\ 56\ \ 0\ 49\ .74$					
$\lambda=\ 56\ 11\ 28\ .10$					
$r''=\ -\ \ \ \ 3\ .54$					
$l=\ 56\ 11\ 24\ .56$	secant	.	.	.	0.2545829
$\Delta p=+0°\ 29'\ 36''.62$	log	.	.	.	3.2495955
$L'=\ 4\ \ 8\ \ 8\ .28$	$\frac{1}{2}(l+l')=56°\ 6'\ 7''.15$			sine	9.9190946
$L=\ 4\ 37\ 44\ .90$ N.	$\Delta z=\ 0°\ 24'\ 34''.65$			log	3.1686901
	$m=57\ 12\ \ 9\ .88$				

Bencampsie bears, or $z=$ S. 57 36 44 .53 E. from Benlomond.

The following are the azimuths reckoned from the south throughout the circle, and the latitudes and longitudes of the preceding stations in pairs, the longitude west being marked +, east −.*

Names of Stations.	Azimuth.	Latitude.	Longitude.
1. Calton Station,	217° 23′ 4″.00	55° 57′ 17″.20	+3° 10′ 46″.68
2. Kellie Law, .	37 43 9.15	56 14 52.76	+2 46 34.73
3. Calton Station,	125° 24′ 23″.72	55° 57′ 17″.20	+3° 10′ 46″.68
4. Bencleugh, .	304 55 14.59	56 11 7.87	+3 45 54.77
5. Bencleugh, .	50° 28′ 58″.27	56° 11′ 7″.87	+3° 45′ 54″.77
6. Bencampsie, .	230 10 31.45	56 0 49.74	+4 8 8.28
7. Bencampsie, .	57° 12′ 9″.88	56° 0′ 49″.74	+4° 8′ 8″.28
8. Benlomond, .	302 23 15.47	56 11 24.56	+4 37 44.90

In the preceding table, the azimuth opposite Calton signifies that Kellie Law bears 217° 23′ 4″.00 from the south towards the west round the circle, and conversely from Kellie Law the Calton bears S. 37° 43′ 9″.15 W., and so on of the rest.

26. In some cases the eastings and westings, and northings and southings, are put down, as already remarked, as co-ordinates, and from these the latitudes, longitudes, and azimuths, are determined. This gives some advantages and some disadvantages, and therefore may or may not be practised at the option of computers. They are tabulated in the following manner, the azimuths being *all* referred to the meridian of Edinburgh.

No.	α	x	Log x.	y	Log y.
1	N. 37° 23′ 4″.00 E.	− 107334.5	5.0307394	+ 82017.3	4.9139053
2	N. 54 35 36.28 W.	− 84782.6	4.9283063	+ 119271.7	5.0765373
3	N. 83 19 2.75 W.	− 22914.0	4.3601014	+ 195571.3	5.2913050
4	N. 73 12 33.28 W.	− 89063.0	4.9496976	+ 295163.3	5.4700623

* In like manner latitudes north may be marked +, south −.

From the co-ordinates in this table the positions may be fixed as before, and all referred to the same meridian. For an exemplification of this, see my Mathematical Tables. To extend right-angled triangles in this manner by parallels to the primary meridian, however, should not be carried too far. To avoid this, a new meridian may be assumed at the distance of every two or three degrees of longitude.

The method of measuring an arc of the meridian may be readily understood from the figure page 41. Suppose the latitude of the point A to be accurately determined, and the azimuths of the sides of the triangle ABD in reference to the meridian XX′, then by the perpendiculars Da, Bb, Cc, &c., the parts A a, $a\,b$, $b\,c$, &c., may be found, the sum of which will be the total arc A h; or by the intersections α, β, γ, δ, &c., the portions A α, $\alpha\,\beta$, $\beta\,\gamma$, &c., may be found, from the sum of which arises A h, the whole arc. Now the latitude of the point h being likewise determined, and the azimuths of the sides hH, hF being also obtained as a verification of those derived from the other extremity at A, the length of the whole arc A h in feet with its corresponding arc in the heavens, the difference of latitude, become known, from which the length of a degree at the middle latitude will be readily found by dividing the extent of the arc in feet by that of its corresponding arc in degrees.

In measuring an arc of the meridian, if the perpendicular I h, fig., page 41, be small, the points I and h may be considered as nearly on the same parallel of latitude; but if it be somewhat considerable, h is not upon the same parallel with I, the difference between which is the small arc of the meridian $\lambda\,l$, fig., page 37, computed from the formula

$$r'' = p''^2\,\tfrac{1}{2}\sin 1''\tan l \quad . \quad . \quad . \quad . \quad . \quad (27)$$

in seconds of arc. This formula may be transformed into another, giving the reduction in feet instead of seconds, and it then becomes

$$r = \tfrac{1}{2} \sin 1'' \,\mathrm{M} \tan l \sin^2 z \,\mathrm{A}^2 \quad . \quad . \quad . \quad . \quad (28)$$

when z and A are given, or when p''^2 is given,

$$r = \tfrac{1}{2} \sin 1'' p''^2 \mu \tan l \quad . \quad . \quad . \quad . \quad . \quad (29)$$

in which μ is the reciprocal of M, readily obtained by using the arithmetical complement of log M in the computation, while p''^2 is the square of the perpendicular arc in seconds, found by a previous part of the computation, and l the latitude.

Constant logarithm $= \log \tfrac{1}{2} \sin 1'' = 4.384545$.

Since the point I is always farther from the equator than the point h, the foot of the perpendicular from it upon the meridian, this correction must be applied to reduce h to the same parallel as I, and must be *added* to the arc of the meridian, when the point I is at the end *nearest* the equator, but subtracted, as in this case, when it is farthest from it.

Examples—1. By the Trigonometrical Survey, volume ii. p. 56, giving an account of the measurement of an arc of the meridian between Dunnose and Clifton, the distance, I h, of the station at Clifton is 4770 feet from the meridian of Dunnose, XX′, on an arc perpendicular to it, in the latitude of 53° 28′ 30″ N. nearly; required the correction $\lambda\, l$, fig., p. 37, of the meridian arc A h, fig., p. 41, in feet?

Here, in reference to formula (28), $l = 53° \, 28' \, 30''$, and $\mathrm{A} \sin z = 4770$ feet;

Whence $\tfrac{1}{2} \sin 1''$, log	4.384545
Log M (Table XIX) to $l = 53° \, 28'.5$.	7.993903
$l = 53° \, 28' \, 30''$ log tangent . . .	0.130395
Log $\mathrm{A}^2 \sin^2 z = 2$ log of 4770 feet . .	7.357036
$r = -0.7343$ foot, log	9.865879

This reduction being required at the end of the arc farthest from the equator, must be subtracted from the arc between the *perpendiculars*, to reduce it to that between the *parallels.*

From one computation, the distance between the perpendiculars, page 55 of Trig. Survey is 1036334.40 feet

By another, page 57, it is . . .	1036333.90
Mean of these two	1036334.15

But the zenith-sector was placed 6.5 feet south of the theodolite station at Dunnose, and 3.5 feet south of the station at Clifton, which increases the arc by 3 feet, their difference; whence, by applying these corrections, we have

Mean value from the triangles		1036334.15 feet
Correction for the positions of the zenith-sector .	+	3.00
Reduction to the parallels	−	0.73
True length of the arc		1036336.42 feet

supposing the trigonometrical computations in the survey to have been accurately performed.

The length stated in the survey, in which this reduction is neglected, is 1036337 feet, that does not differ materially from the preceding result, on account of the smallness of the perpendicular arc, and for that reason was probably omitted.

2. The same omission in the measurement of the French arc of the meridian between Montjouy and Formentera would have produced an error of 169.88 French toises, on account of the magnitude of the perpendicular arc from Formentera on the meridian of Paris, had it not been counteracted partly by an opposite error, arising from the insufficiency of a formula of Delambre, as there employed, which is given in the third volume of the *Base du Système Métrique*, page 4, illustrated by a numerical example, page 190, producing an

error of 100.07 toises, with a *contrary sign.* The difference of these two make on the whole an error of 69.81 toises in the results of the original commission, published in the *Connaissance des Tems* for 1810. This error was first detected by M. Puissant, and has been finally verified by a new commission lately appointed for that express purpose. This correction ought, therefore, hardly in any case to be neglected. M. Puissant, however, shews that Delambre's formula* is quite accurate when the convergence of the meridians, c'', is taken into account, or if, instead of z, the azimuth simply, $(z+c'')$ be employed. It is certain that Delambre did, in some instances, understand the formula in this sense; but it appears probable that he had not attended to it in some manuscript instructions communicated by him to the commission of 1808.

To avoid any difficulty from this cause, it may be recommended, in general, to trace the meridian arc through a continued series of triangles, so that the extremities of that arc may commence and terminate in the vertices of the first and last triangle, as nearly as may be convenient. If not, the small correction derived from this formula must not be neglected.

It would extend this paper too much to enter at length into this subject, which may be seen more fully developed in the Introduction to my Mathematical Tables.

In *Marine Surveying*, a table of meridional parts is generally required to one or two places of decimals, and in cases of great accuracy they should be used for a spheroid of about $\frac{1}{300}$ of compression. If the reduction of the latitude from Table XVI. be subtracted from the apparent or observed latitude, the meridional parts answering to the

* $dl = -\text{A}\cos z + \frac{1}{2r}\text{A}^2\sin^2 z\tan l + \frac{1}{6r^2}\text{A}^3\sin^2 z\cos z(1+3\tan^2 l)$ in feet.

remainder or geocentric latitude, will be those on the spheroid. I generally, however, prefer the following formula, in which the first term gives the meridian parts on the sphere, and the remaining terms give the corrections to reduce the meridian parts on the sphere to those on the spheroid of $\frac{1}{300}$ of compression. Let P = the meridian parts on the spheroid to l, the observed latitude, then

$$P = 7915'.705 \log \{\log \tan (45^\circ + \tfrac{1}{2} l) - 10\} \\ - 22'.9182 \sin l + 0'.0127 \sin 3l - \&c. \quad . \quad . \quad (30)$$

Log 7915.705 = 3.8984896, log 22′.918 = 1.360181, and log 0′.0127 = $\bar{2}$.10380.

Example 1. Required the meridian parts to latitude 55° 30′, both on the sphere and spheroid?

1. Constant log . .	3.8984896	2. Constant log .	−1.360181
$45^\circ + \frac{1}{2} l = 72^\circ\ 45'$.	9.7058012	$\sin l =$ . .	+9.915994
1st term = +4020′.60 log	3.6042908	2d term −18′.89 log	−1.276175
2d term = − 18.89			
3d term = + 0.00		3. Constant log .	+8.10380
		Sin 3 l . .	+9.36818
P′ . = 4001.71			
		3d term + 0.003 log	+7.47198

Hence the meridian parts for latitude 55° 30° are 4020′.60 (the first term) on the sphere, and 4001.71 on the spheroid of $\frac{1}{300}$ of compression.

Ex. 2. Required the meridian parts for latitude 56° 30′?

Ans.	On the sphere	4127.90
	On the spheroid, or P = ...	4108.80

In this way the meridian parts may be computed to every degree and minute throughout the extent of the survey. Now, when the proper scale is chosen for a degree of longitude, the differences of the meridian parts for each degree, &c., throughout the extent of the survey from the same scale, will give the graduation of the scale of latitude. Thus, $P - P' = 4108'.80 - 4001'.71 = 107'.09 = 1^\circ 47'.09$ to be taken from the scale selected for longitude, to give the extent of a degree of latitude, between the latitudes 55° 30′ and 56° 30, on the terrestrial spheroid.

In Naûtical Surveying it is sometimes convenient or necessary to find the distance of a point near the horizon of the sea by its observed depression. An imperfect solution of this problem is given in Horsburgh's edition of Mackenzie's Marine Surveying, section iv., by considering the triangle *right angled*, and omitting the effects of terrestrial refraction.* For this purpose let H be the *obtuse angle* near the horizon, formed by the line from the eye of the observer to that point, and another line from the centre of the earth to the same point, D the observed depression, M the logarithmic modulus, r the earth's radius, and h the height on which the depression is taken, then making $\log \frac{M}{r} = \bar{8}.317198$, when r is the mean radius of the earth, the following formulæ may be readily investigated.

1. $\text{Log} \sin H = \log \cos D + \frac{Mh}{r}$.
2. $\frac{1}{2}\{D - (H - 90^\circ)\} = A''$; also $a'' = 0.84\,A$ and $\alpha'' = \frac{1}{5}\,a''$.
3. $\text{Log}\,K = \log \text{cosec}\,(D - a'') + \log \cos\,(D + \alpha'') + \log h$.
4. $\text{Sin}\,\frac{1}{2}H = \{1 \pm \sec D_{,} \sin (D + D_{,}) \sin (D - D_{,})\}^{\frac{1}{2}}$.

In the last formula $D - D_{,}$ is the difference of the depression of the given point D and that of the horizon $D_{,}$, which difference may be measured with a sextant, while the depression of the horizon may be computed by the usual formulæ, and then $D_{,}$ and $D - D_{,}$ become known without the

* There appears to be an error committed in the operation or solution of the example, making the distance $7\frac{3}{4}$ miles, instead of 4741 feet, or about $\frac{9}{10}$ of a mile only!

employment of an altitude-circle on shore, and in this case the upper sign must be used.

Example. Let the observed depression of a given point from the top of Goatfell, in the island of Arran, D=2° 18′ 8.″4 by an astronomical circle, and the height h of the circle above the level of the sea 2861.5 feet; required K, the chord measuring the distance of the point observed from the station on which the observation was taken?

Constant log	. $\bar{8}$.317198		
h=2861.5 feet, log	3.456594		
Sum . .	$\bar{5}$.773792	Natural number	0.0000595
D= 2° 18′ 8″.4		cosine . . .	9.9996493
H=92 5 52 .4		sine . . .	9.9997088

D−(H−90°)= 12 16 .0
Half= 6 8=A″=368″, and 368″ × 0.84=0° 5′ 9″.12=α″
$\frac{1}{5}$ α″=0 1 1.82=α″

D= . . .	2° 18′ 8″.4	. . .	2° 18′ 8″.4
α″= . . .	− 5 9.1,	α″= .	+ 1 1.8
D−α″= . . .	2 12 59.3,	D+α″=	2 19 10.2

D−α″=2° 12′ 59″.3	cosecant . .	1.4125687
D+α″=2 19 10 .2	cosine . .	9.9996440
h= 2861.5 feet,	log . . .	3.4565938
K=73927.6 feet,	log . . .	4.8688065

the first approximation, which, in moderate distances, will in general be sufficient.

For a second approximation, the following method may be employed, in which log K is that previously found.

5. Log α″=Const. log 7.617089+log K.

6. Log α″=Const. log 6.896930+log K.

Const. logs,	1st	7.617089,	2d . . .	6.896930
Log K . .		4.868806	. . . .	4.868806
α″=5′ 6″.1 log .		2.485895,	α″=58″.3 log	1.765736

$D = 2^\circ\ 18'\ 8''.4$	. . $2^\circ\ 18'\ 8''.4$,	$D - \alpha''$ cosecant	1.412406
$\alpha'' = -\ 5\ 6.1$	$\alpha'' = +\ 58.3$,	$D + \alpha''$ cosine	9.999644
$D - \alpha'' = 2\ 13\ 2.3$,	$D + \alpha'' = 2\ 19\ 6.7$	h log . .	3.456594
$K = 73900$ feet	. . . .	log . .	4.868644

Another repetition, using this last value of K, would not produce any sensible change in its value, and it may therefore be reckoned correct.

In making use of formula (4), we suppose $D - D_{\prime}$, the angle between the point whose distance is required and the visible horizon, to be measured with a sextant, and found to be 1° 25° 29″.7.

To compute the dip we have

Constant logarithm		. . . .	1.771208
$h = 2861.5$ feet,	$\frac{1}{2}$ log	. . .	1.728297
$D_{\prime} = 0^\circ\ 52'\ 38''.7$	log	. . .	3.499505
$D - D_{\prime} = 1\ 25\ 29.7$	by observation, sine	.	8.395623
$D = 2\ 18\ 8.4 =$ sum			
$D + D_{\prime} = 3\ 10\ 47.1$	sine	. . . .	8.744047
			17.139660
Nat. N. 0.037152	log	. . .	8.569830
1.0			
Sum = 1.037152			
Half = 0.518576	log	. . .	19.714812
$\frac{1}{2}$ H = 46° 3′ 52″.5	sine	. . .	9.857406
2			

$H = 92\ 7\ 45.0$	$D - \alpha'' = 2^\circ\ 12'\ 56''.7$	cosec	1.412710
$H - 90^\circ = 2\ 7\ 45.0$	$D + \alpha'' = 2\ 19\ 10.7$	cos	9.999644
$D = 2\ 18\ 8.4$	$h = 2861.5$ feet,	log	3.456594
$D - (H - 90^\circ) -\ 10\ 23.4$	$K = 73952$	feet, log	4.868948
Half $=\ 5\ 11.7 = \alpha''$			
$\frac{1}{5}\alpha'' =\ 1\ 2.3 = \alpha''$			

Another repetition, as shewn in last example, gives K = 73900 feet, the true value, as before.

The former method is recommended when the observer has a good altitude and azimuth circle; the latter, when he has a sextant or reflecting circle only, as is frequently the case with nautical surveyors.

27. Trigonometrical Levelling is an operation which generally accompanies Trigonometrical Surveying, because the exact situation of a given point on the earth's surface is accurately fixed by the three co-ordinates, latitude, longitude, and elevation above the mean level of the sea. The triangle formed in a vertical plane above the earth's surface in this operation is called a *hypsometrical triangle.* It is formed by the chord of the terrestrial arc comprised between the verticals of the two stations where the reciprocal zenith distances have been observed, by the straight line which joins the two points of observation and the difference of level dh. When reciprocal and simultaneous observations are made, that is, when observations are made from two different points on one another at the same time, the results are esteemed the most accurate, as the effects of refraction at the two places is determined by observation. In this case, let δ be the observed zenith-distance at the one place, and δ' that at the other, less than the former, while C is the angle contained by two vertical lines drawn from the surface of the earth to its centre; then the difference of altitude will be found by the following formulæ.

$$dh = K \sin \tfrac{1}{2}(\delta - \delta') \sec \tfrac{1}{2}(\delta - \delta' + C) \quad . \quad . \quad . \quad . \quad (1)$$

$$= K \tan \tfrac{1}{2}(\delta - \delta') \text{ very nearly.} \quad . \quad . \quad . \quad . \quad (2)$$

But it frequently happens that reciprocal and simultaneous observations cannot be observed; then, in that case, if n be the coefficient of terrestrial refraction,

$$dh = K \sec \tfrac{1}{2} C \cot \{\delta + (n - 0.5) C\} \quad . \quad . \quad . \quad . \quad (3)$$

$$= K \cot \{\delta + (n - 0.5) C\} \text{ very nearly.} \quad . \quad . \quad . \quad (4)$$

The same thing may be done by the following formula, in which the first part is the solution of a right-angled plane triangle, and the second contains the effects of the curvature of the earth combined with the refraction.

$$dh = \mathrm{K} \cot \delta + \frac{\mathrm{K}^2}{\varrho}(0.5 - n) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which ϱ is the radius of curvature equal to the mean radius of the earth nearly, or exactly $\frac{1}{\varrho} = f \sin 1''$, in which f is the factor, from Table XIX., to convert feet into seconds of arc.

To determine the height of the point of observation by the observed depression of the horizon of the sea,

$$dh = \tfrac{1}{2}\varrho(1+n)^2 \tan^2(\delta - 90^\circ) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in which $\delta - 90^\circ$ may be replaced by D, the observed depression of the horizon of the sea,

$$dh = \tfrac{1}{2}\varrho(1+n)^2 \tan^2 \mathrm{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where $\frac{1}{2}\varrho = \frac{\frac{1}{2}\mathrm{R}''}{f}$, as before.

For many practical purposes the mean value of $n = 0.08$ of the intercepted arc will be sufficient, and

$$dh = 0.5832\,\varrho \tan^2 \mathrm{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

or, when the depression D does not exceed a few minutes,

$$dh = 0.5832\,\varrho \sin^2 1'' \, \mathrm{D}''^2 \text{ nearly} \quad . \quad . \quad . \quad . \quad (9)$$

Constant log. of $0.5832\,\varrho \sin^2 1'' = 6.457582$.

The same value of n might be introduced into formulæ (4) and (5), and the former would become

$$dh = \mathrm{K} \cot(\delta - 0.42\,\mathrm{C}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

the latter becomes

$$dh = \mathrm{K} \cot \delta + \mathrm{K}^2 . \frac{0.42}{\varrho} \quad . \quad . \quad . \quad . \quad (11)$$

and $\log \frac{0.42}{\varrho} = \bar{8}.302632.$

These mean coefficients are, however, combined properly in Tables XXI. and XXII., corresponding to formulæ (4) and (9). By a paper in the Edinburgh New Philosophical Journal for April 1841, I have given the following formula to compute the value of n in given circumstances, and the computation may be readily performed by the aid of Table XI. and the auxiliary refraction tables.

$$n = \frac{a\,r}{2\,B\,l} \cdot b \left(\frac{1}{1+\beta(t-50^\circ)} \right)^2 \cdot \frac{1}{1+\beta'(\tau-50^\circ)} \left(0.75 - \frac{f}{3\,b} \right) \quad (12)$$

$$\text{Log}\, \frac{a\,r}{2\,B\,l} = \text{Constant logarithm } 7.57877.$$

This const. log. is combined with the factor $\left(0.75 - \frac{f}{3\,b}\right)$ in Table XI., for the use of which see the explanation of the tables.

1. To exemplify these formulæ, we find that at

	° ′ ″
Clermont Ferrand $\delta' =$ . . .	83 33 33.37
Puy de Dôme δ	96 30 38.67
$\delta - \delta'$	12 57 5.30
$\frac{1}{2}(\delta - \delta') =$	6 28 32.65

Also $b' = 28.839$ in., $\tau' = 52°.7$ F. $t' = 45°.14$ F.
$b = 25.383$, $\tau = 51.1$ F. $t = 48.92$ F.

The base K in English feet log . .		4.4875708
To $\frac{1}{2}(l + l') = 45° 47'$ and $\alpha = 84\frac{3}{4}°$ log 0		7.9930870
C = 5′ 2″.45 . . log		2.4806578
$\delta - \delta' = 12° 57$ 5.30		
$\delta - \delta' + C = 13$ 2 7.75		
$\frac{1}{2}(\delta - \delta' + C) =$ 6 31 3.87	secant . .	0.0028161
$\frac{1}{2}(\delta - \delta') =$ 6 28 32.65	sine . . .	9.0522415
K . . .	log . . .	4.4875708
$d\,h = 3488.42$ feet	log . . .	3.5426284

the result by reciprocal and simultaneous observations, which is reckoned the most correct method; but as we have the state of the barometer and thermometer recorded, we shall compute the same difference of level by formula (3), requiring the calculation of the value of n by Table XI.

2. Log to $b=25.383$ in. and $t=49^{\circ}$ nearly .	7.45114
$\tau=51.1$ log (Table VII.) .	9.99996
$t=48^{\circ}.92$ log $\times 2$ (Table VIII.)	0.00192
$b=25.383$ log	1.40454
$n=0.07204$ log	8.85756
Log C, as above, ...	2.4806578
$n-0.5=-0.42796$ log . . .	$-\bar{1}.6314032$
$v'=-0^{\circ}\ 2'\ 9''.44$ log . . .	-2.1120610
$\delta=96\ 30\ 38.67$	
$\delta_{,}=96\ 28\ 29.23$ cotangent .	-9.0549570
$\frac{1}{2}C=0\ 2\ 31.22$ secant . .	$+0.0000001$
Log K	$+4.4875708$
$dh=-3487.61$ feet log . . .	-3.5425279

From the sign $-$, this shews that Clermont Ferrand is 3487.6 feet under the summit of Puy de Dôme. We shall also determine the elevation of Puy de Dôme above Clermont Ferrand.

3. Log from Table XI. to $b'=28.339$ in. and $t'=45^{\circ}$	7.45175
$\tau'=52^{\circ}.7$ F. log (Table VII.) . .	9.99988
$t'=45^{\circ}.14$ F. log $\times 2$ (Table VIII.) .	0.00884
$b'=28.839$ log	1.45998
$n'=0.08327$ log	8.92045
$-0.5=-0.5$	
$n-0.5=-0.41673$ log	$-\bar{1}.6198548$
Log C, as before, ...	2.4806578
$v'=-0^{\circ}\ 2'\ 6''.05$ log . . .	-2.1005126
$\delta'=83\ 33\ 33.37$	
$\delta_{,}=83\ 31\ 27.32$ cotangent .	$+9.0550219$
$\frac{1}{2}C=0\ 2\ 31.22$ secant . .	0.0000001
Log K	4.4875708
$dh=3488.13$ feet, log	3.5425928

Agreeing almost exactly with the first solution by reciprocal and simultaneous observations, while, by the value of n computed from the formula, the results differ by about half a foot only,—a strong proof of the accuracy of the principle which I employed in its investigation.

4. To determine the coefficient of refraction by observation, we have $r=n\,C$, C being the intercepted arc, and n the effect of refraction, a part of that arc.

But $$r=\tfrac{1}{2}\,C-\tfrac{1}{2}\,(\delta+\delta'-180^\circ).$$

Introducing this into the preceding equation, and it becomes by reduction,

$$n=\frac{180^\circ+C-(\delta+\delta')}{2\,C} \quad . \quad . \quad . \quad . \quad (13)$$

Now at Puy de Dôme.	$\delta=$		$96^\circ\ 30'\ 38''.67$
Clermont Ferrand	$\delta'=$		83 33 33.37
$\delta+\delta'=$	.. . . .		180 4 12.04
$180^\circ+C=$	. . .		180 5 2.45
$180^\circ+C-(\delta+\delta')$	. .		0 0 50.41
$0'\ 50''.41$	log	. . .	1.7025167
$2\,C=10\ \ 4.90$	log	. . .	2.7816836
$n=0.083336$	log	. . .	$\bar{2}.9208331$
-0.5			
-0.416664	log	. .	$-\bar{1}.6197860$
Log C, as before,		. . .	2.4806578
$v'=-0^\circ\ 2'\ 6''.02$	log	.	-2.1004438
$\delta=$ 96 30 38 .67			
$\delta_{\prime}=$ 96 28 32 .65	cotangent		-9.0550213
K	log	. .	4.4875708
dh 3488.13 feet	log	.	-3.5425921

Here the result is the same as before. As v' here found agrees almost exactly with that previously determined from the computed value of n, the computation need not be repeated. In fact, the distance is too small for the refraction to produce a very marked effect, though this example has been selected by both Puissant and Biot to test their formulæ. From this, too, it appears that the refraction determined by observation is not that at either point, but a mean between them.

I shall now proceed, from Inchkeith, to determine the heights of the former points (in the preceding part of this paper) above the sea.

5. At Inchkeith, in August 1840, I found the zenith distance of the summit of the dome of Edinburgh Observatory to be 89° 40′ 24″.45, at the distance of 30117 feet, bearing S., 18° 43′ 7″.24 W., when the barometer b stood at 29.675 in., and Fahrenheit's thermometer at 63°.8; what was the height of the summit of the Dome above the place of observation, and above the mean level of the sea?

Log from Table XI. to b and t . .	7.44996
$\tau=63°.8$ log from Table VII. . .	9.99941
$t=63.8$ 2 log from Table VIII. . .	9.97538
$b=29.675$ log	1.47239
$n=$ 0.07892 log	8.89714
-0.5	
$n-0.5=-0.42108$ log	-9.624364
Log O for lat. 56° and $z=19°$ . .	7.993624
$A=30117$ feet, log	4.478812
$\delta'=-0°\ 2'\ 4''.97$ log . . .	-2.096800
$\delta=$ 89 40 24.45	
$\delta_{,}=$ 89 38 19.48 cotangent .	$+7.799698$
$A=30117$ feet, log	4.478812
$dh=189.89$ feet, log	2.278510

$h=$ 4.00 = height of circle above ground.
$h'=175.00$ = height of ground above high-water.
$h''=$ 8.50 = half rise of tide.

$H'=377.39$ feet.

H′ = 377.39 feet, the height of the summit of the dome above mean tide.
Station 26.90 feet under the dome.

H = 350.49 feet, the height of the axis of the circle on the cylindrical stone south-west of the Observatory, from which I took the following observations on Bencleugh.

6. From this point the summit of Bencleugh was observed to have a zenith-distance of 89° 22′ 41″.85, when the barometer stood at 29.68 in. and Fahrenheit's thermometer at 62°, the middle latitude being 56° 4′ 12″ N., bearing N. 54° 36′ W., distant 146334.6 feet; required the height of Bencleugh above the place of observation, and also above the mean level of the sea?

To $b = 29.68$ and $t = 62''$	log (Table XI.)	7.45022
$\tau = 62°$	log	9.99948
$t = 62°$	log × 2 . . .	9.97854
$b = 29.68$	log	1.47246
$n =$ 0.07956	log	8.90070
−0.5		
	log A	5.1653471
$n - 0.5 = -0.42044$	log . . .	−9.6237040
Log O to 56° and 54°.6	. . . .	7.9931162
$\delta' = -0° 10' 5''.57$	log . . .	−2.7821673
$\delta =$ 89 22 41 .85		
$\delta_{,} =$ 89 12 36 .28	cotangent . . .	8.1394892
A = 146334.6	log	5.1653471
$m h$, from Table XXIV.	. . .	+ 73
S, from Table XXIV.	. . .	+ 19
$d h = 2017.65$ feet,	log	3.3048455
$h =$ 350.49 = height of Calton Station.		
H = 2368.14 = height of Bencleugh.		

In this manner the elevation of any number of points may be determined successively.

The difference of level may be found approximately with sufficient precision for many purposes by reciprocal zenith-distances, independent of triangulation. This method is not so accurate as the preceding, but will frequently be useful where tolerable accuracy only is necessary.

From a simple investigation, when $n=0.08$, it will be found that the sum of the refractions, or

$$r+r'=\tfrac{1}{8}\,(\delta+\delta'-180^\circ)\ \text{nearly.} \quad . \quad . \quad . \qquad (14)$$

where δ and δ' are the apparent zenith-distances. If C be the true angle at the centre, and c the apparent,

$$C=c+r+r' \quad . \quad . \quad . \quad . \qquad (15)$$

in which $r+r'$ is got from formula (14), and $c=\delta+\delta'-180^\circ$.

If, however, n and n' be computed from the state of the barometer and thermometer, as has been already shewn, then

$$C=\frac{c}{1-(n+n')} \quad . \quad . \quad . \quad . \qquad (16)$$

This value of C will generally be more accurate than the preceding. The difference of level will then be computed by the following formula,

$$d\,h=2\,\varrho\tan\tfrac{1}{2}\,C\tan\tfrac{1}{2}\,(\delta-\delta')+2\,\varrho\tan^2\tfrac{1}{2}\,C\tan^2\tfrac{1}{2}\,(\delta-\delta') \quad . \qquad (17)$$

in which the first term will generally be sufficient.

7. Let $\delta=90^\circ\ 25'\ 43''.11$, $b=28.355$ in., $\tau=44^\circ.4$, $t=44^\circ.4$ F.
$\delta'=89\ 54\ 41.22$, $b=29.339$, $\tau'=55.4$, $t'=52.7$ F.

		From these,	
$\delta+\delta'-180^\circ=c=$	20 24.33	$n=$	0.08220
$\tfrac{1}{5}\,c=$	4 4.87	$n'=$	0.08195
$C=c+\tfrac{1}{5}c=$	24 29.20	$n+n'=$	0.16415
			1.0
$\delta-\delta'=$	31 1.89		
$\tfrac{1}{2}\,(\delta-\delta')=$	15 30.94	$1-(n+n')=$	0.83585

Log $c''=\log 20'\ 24''.33$	. . .		3.08790
Log $1-(n+n')=$ 0.83585	. .	(subt.)	$\bar{1}.92113$
C $=24'\ 28''.13$	. .	log	3.16677

Logarithm of 2 R″	5.6154551
$a \cdot c \cdot \log O$ to $\frac{1}{2}(l+l') = 38°\ 52'$ and $\alpha = 19°\ 47'$	2.0052030
$\frac{1}{2}C = 0°\ 12'\ 14''.06$ tangent	7.5513083
$\frac{1}{2}(\delta - \delta') = 0\ \ 15\ \ 30\ .94$ tangent	7.6544995
$dh = 670.6$ feet, log	2.8264659

From other data the value of $dh = 675.5$, exceeding the preceding by 4.9 feet only.

The approximate distance may also be obtained by adding to

Log C	3.16677
$a \cdot c \cdot \log O$ . . .	2.00520
A = 148580 feet, log . .	5.17197

8. By an astronomical circle, the axis of which was 3.5 feet above the rock, the depression of the horizon of the sea from the summit of Dunii, in the Island of Iona, was 17′ 52″, bearing about S. 70° W.; required the height of Dunii, the highest hill in Iona?

To lat. 56° 20′ N. and $\alpha = 70°$ log O″	6.458479
$D'' = 17'\ 52'' = 1072''$, $\log \times 2 =$ . .	6.060390
$dh = 330.2$ feet, log	2.518869
$h' = -3.5 =$ height of circle.	
H = 326.7 feet, the height of the ground.	

This method is sufficient for most cases. However, as circumstances will occur where the greatest possible accuracy may be required, then $\frac{1}{2}\rho$, from Table XIX. $= \frac{\frac{1}{2}R''}{f}$, in which f is the factor to convert feet into seconds, will give, when combined with $(1+n)^2 \tan^2 D$, the height, with all the accuracy that can be expected.

In Marine Surveying, it is seldom convenient, and often impossible, to determine the direction of the meridian by the pole-star, as has been shewn in the preceding pages; and in the practice of ordinary Surveying, such a degree of precision is unnecessary. In this case recourse may be had to the methods recommended in pages 28, 29, &c., as illustrated by the following examples.

Example 1. On the 10th of July 1837, at 7^h A. M., in latitude 7° 31′ 20″ S., and longitude 153° 10′ E., the observed altitude of the sun's lower limb was 10° 13′ 0″, and at the same instant the observed distance of the sun's nearest limb from a well-defined point of land on the same level with the eye to the left of the sun was 95° 16′ 0″. The index-error of the former sextant was −0′ 50″, that of the latter +1′ 10″, the height of the observer's eye taking the sun's altitude being 14 feet; required the true bearing of the point of land, and the variation of the compass, when the magnetic bearing of the same point was N. 5° 10′ W.?

	h.	m.	s.		°	′	″
Ship time, July 9,	19	0	0	To G. M. T. Sun's P. D.	112	20	16
Longitude in time	10	12	40 E.	Sun's semidiameter		15	45
Greenwich Mean T.	8	47	20				

	°	′	″		°	′	″
Obs. Alt. l. l. .	10	30	0	Observed distance .	95	16	0
Index error .	−	0	50	Index error . .	+	1	10
Dip to 14 feet .	−	3	43	Sun's semidiameter .	+	15	45
Semidiameter .	+	15	45	Apparent central distance	95	32	55
Apparent altitude	10	41	12				
Correction .	−	4	56				
True altitude .	10	36	16				

Now by the rule of the circular parts of Napier, applied to the right-angled spherical triangle HO☉, fig. page 28,

$$R \times \cos H\odot = \cos \odot O \times \cos HO, \text{ or } \cos HO = \cos H\odot \sec \odot O.$$

H☉, or apparent distance,	95°	32′	55″	cosine	8.985383
☉O, or apparent altitude,	10	41	12	sec	0.007599
H☉, or HZ☉,	95	38	49	cos	8.992982

the difference of the azimuths of the sun and the object.

The sun's true azimuth may be computed by a formula similar to that for time, in page 20, thus:

To find the Azimuth.

Rule. Set down the polar distance, the true altitude, and the latitude, then find half their sum, and the difference between this half sum and the polar distance. To the log secant of the altitude add the log secant of the latitude, the log cosine of the half sum and the log cosine of the difference; half the sum of these four logarithms will be the log sine of half the azimuth from the meridian, to be reckoned from the *south* in *north* latitude, and from the north in south latitude.

	°	′	″		
Polar distance . . .	112	20	16		
True altitude . .	10	36	16	secant	0.007481
Latitude . . .	7	31	20	secant	0.003754
Sum . . .	130	27	52		
Half . . .	65	13	56	cosine	9.622153
Difference . . .	47	6	20	cosine	9.832924
					19.466312
Half . . .	32°	44′	54″	sine	9.733156
			2		
Sun's true bearing	N. 65	29	48	E.	
Object to left of sun .	95	38	49		
True bearing of object	N. 30	9	1	W.	
Magnetic bearing	N. 5	10	0	W.	
Variation of compass .	24	59	1	W.	

Ex. 2. On the 1st of May 1834, in latitude 33° 8′ 0″ N., longitude 16° 10′ W., the height of the eye 18 feet, the following observations were made to determine the true bearing.*

Mean time 9h 35m 52s A.M.	Obs. Alt. ⊙ 52° 25′ 30″	Obs. Dist. ⊙	111° 34′ 0″
Longitude 1 4 40 W.	Dip to 18 ft − 4 12	S. D. .	+ 15 53
	Sun Dr. + 15 53		
Red. G.T. 10 40 32		App. Dist.	111 49 53
	App. Alt. 52 37 11		

* The marks ⊙ mean the sun's lower limb, and ⊙ shews the position of the sun in the cross wires of the telescope, &c.

	° ′ ″	
App. Alt.	52 37 11	
Correction	— 39	☉'s Pol. Dist. 75° 0′ 10″
True Alt.	52 36 32	

	° ′ ″		
Polar distance	75 0 10		
True altitude .	52 36 52	secant . .	0.216631
Latitude . .	33 8 0	secant . .	0.077067
Sum . .	160 44 42		
Half . .	80 22 21	cosine . .	9.223345
Difference .	5 22 11	cosine . .	9.998090
Sun's bearing	S. 69 48 40 E.	Reduced versine	9.515133

	° ′ ″		
Apparent altitude .	52 37 11	secant .	0.216738
Apparent distance .	111 49 53	cosine .	9.570399
Z to the right .	127 46 25	cosine .	9.787137
Sun's bearing .	S. 69 48 40 E.		
True bearing of object	S. 57 57 45 W.		

Should the object in Example 1. be not on the level of the eye, the following method of computing the angle HZ☉ must be employed.

	° ′ ″		
App. Cent. Dist. (D)	95 32 55		
Z. D. of point observed	90 0 0	cosecant .	0.000000
Sun's apparent Z. D.	79 18 48	cosecant .	0.007600
Sum . . .	264 51 43		
Half (H) . .	132 25 51.5	sine .	9.868110
Difference =H−D=	36 52 56.5	sine .	9.778277
			19.653987
Half . . .	47 49 24.5	cosine .	9.826993
	2		
Angle at zenith .	95 38 49	as before,	

and this plan must be always followed when both zenith-distances differ considerably from 90°, or even when it is *doubtful* if the object be on the same level with the eye.

Ex. 3. At Dunii Cairn, Iona, on the 21st of August 1839, in latitude 56° 20′ 34″ N., longitude in time 25ᵐ 34ˢ W., observations were taken with an astronomical circle, and reduced as stated below.

	h.	m.	s.
Mean time by watch	7	9	46.2
Error of watch fast	−	11	54.0
Mean time	6	57	52.2
Equation of time	−	3	0.2
Apparent time, or angle at the pole, t .	6	54	52.0

	°	′	″		
Polar distance .	77	49	2		
True altitude .	2	42	16	secant	0.000484
Latitude . .	56	20	34	secant	0.256315
Sum . .	136	51	52		
Half . . .	68	25	56	cosine	9.565377
Difference . .	9	23	6	cosine	9.994148
					19.816324
	54	2	13.5	sine	9.908162
			2		
Azimuth .	S. 108	4	27 W.		
	180	0	0		
Azimuth .	N. 71	55	33 W.		

The azimuth may also be determined by the formulæ in page 27.

	°	′	″
Let l = the latitude . .	= 56	20	34 N.
and d = the declination . .	= 12	10	58 N.
$d+l$ = ...	68	31	32
$\frac{1}{2}(d+l)$ = ...	34	15	46
$l-d$ = ...	44	9	36
$\frac{1}{2}(l-d)$ = ...	2	4	48

$\frac{1}{2}t$ = 3h 27m 26s cot	9.895022 .	. . . .	9.895022
$\frac{1}{2}(l-d)$ = 22° 4′ 48″ cos	9.966920 sine	. . . .	9.575073
$\frac{1}{2}(l+d)$ = 34 15 46 cosec	0.249500 sec	. . . .	0.082776
$\frac{1}{2}(m+e)$ = 52 16 18.5 tan	10.111442	$\frac{1}{2}(m-e)$ = 19° 39′ 18″.6 tan	9.552871
$\frac{1}{2}(m-e)$ = 19 39 18.5			

m = N. 71 55 37.0 W.		Change of azimuth in	
1 . z = 108 4 27.0		10 minutes .	2° 3′ 54″.0
2 . z = S.108 4 23.0 W.		1 minute .	12 23 .4
		10 seconds .	2 3 .9
		1 second .	12 .39

Mean = S.108 4 25 W.
Arc = 68 54 3 or horizontal angle at Dunii by the circle to Carn Cul ri Eirn.
S. 39 10 22 W. the bearing of Carn Cul ri Eirn from Carn Dunii in Iona.

Ex. 4. On the 11th of August 1841, at my station, in latitude 55° 27′ 56″.74 N. longitude, 4° 37′ 35″ W., at $4^h\ 40^m\ 17^s.5$ mean time, by observations on the limbs of the sun, taken alternately above and below the central horizontal wire, and to the right and left of the vertical wire, in opposite quadrants of the diaphragm, so that the mean of all might be that of the sun's centre at the intersection of the wires in the centre of the telescope, I found the true altitude of the sun's centre, deduced from the *vertical arc* of my circle, to be 24° 25′ 45″, when the polar distance was 74° 47′ 14″; while, by the *horizontal arc*, the angle between the sun's centre and Ayr High Spire, was 2° 52′ 23″.3 W., distant 1152 feet; required the latitude and longitude of the spire?

		°	′	″	
Ans. The azimuth of the sun . .	S.	81	16	36.8	W.
... ... spire . .	S.	84	9	0.1	W.
Reduction of station to spire, in lat. .	—	0	0	1.16	S.
... ... , in long.	+	0	0	19.88	W.
Latitude of spire . . .		55	27	55.58	N.
Longitude of spire . . .		4	37	54.88	W.

In a similar way the common theodolite may be employed, and the result will be within a few minutes of the truth. Whence also the variation may be obtained by taking bearings at the same time by the needle.

RAILWAY SURVEYING.

1. In the present times, when railways are constructing in all parts of the United Kingdom, as well as abroad, on account of their great importance and general utility, a recommendation in favour of their adoption cannot now be necessary. However evident this general proposition may be, yet it requires much caution and considerable professional knowledge, to select the lines in such a manner as to enable the country, as well as the public and shareholders, to derive the full advantage they are certainly calculated to produce. The surveyor undertaking such a work ought first to ascertain, by personal observation, the nature of the country through which it is intended to pass, with regard to its localities, its structure, and geological character. This might lead him to the choice of several lines apparently equally favourable, as far as a cursory inspection of the ground by the eye, chiefly through the medium of its lakes, rivers, and mountain-ranges, could determine.

2. In the selection of railways, too, the amount of traffic, to a certain extent, ought to regulate the nature of the construction. If there is a certainty of great traffic, the expenditure in tunnelling, cutting, embanking, and viaducts, may be very considerable, in order to improve the gradients; but if a moderate trade only is to be expected, such an expenditure must be injudicious, because it increases the charges, or diminishes the profits, of the original share-

p 1° 32 . . . . . 9.895092

t 19h 10 . . . . . 9.5750

. . . . . 0.0827

u − 0° 28

= 19° 39′ 18″.6 tan 9.5526

λ 56 30

Change of azimuth in

l 56 1 . 2° 3′ 54″.0

. 12 23.4

. 2 3.9

. 12.3

In the sa

computed

But sin

distance a

the arc d

tions, as i

correction

dl =

dm =

in which

$p'' = 1° 32'$

sin 1″

p'' sin 1″

$l = 56° 1'$

p'' sin 1″

1st Obser

2d Obser

Mean

Mean − 1

Log V′ fo

Log f

$t = 19^h$ 1

p'' sin 1″

$dl = +0$

In this

series,

[illegible]

holders, and thus, unless at the expense of the public, they must receive an inadequate remuneration for their capital. Circuitous lines, to serve inferior provincial towns, are not to be recommended, except to a limited extent, because the greatest amount of the whole traffic may be expected from the large towns at the extremities, for whose use chiefly the railway would be constructed, and through whose influence the bill for such a purpose must chiefly be carried.

The passengers between these towns would thus be compelled to pay fares for these additional miles so thrown into the railway, while, at the same time, the duration of transit would be increased, by this means entailing a positive loss on the majority of the passengers, both in money and time. It would generally, therefore, be better to connect these inferior towns with the main line by short branches.

The advantages, therefore, to be derived from the use of railways, are, rapidity of transit, and economy of charge; to accomplish which, the following principles should be kept constantly in view:—

1°, One of the conditions which must not be departed from in laying out great lines of railway is, that those lines may be traversed throughout their whole extent by locomotive engines; and, in order to avoid, as much as possible, interruptions and delays, that the same engine should draw the same train.

2°, Another condition is, to diminish, as far as practicable, the time of transit between two given points, by reducing the length of the railway. In this case the straight line, either horizontal or having one uniform slope, will be the most advantageous. It is this line which ought to be selected, or the nearest practicable one to it, both horizontally and vertically.

3°, If two lines may be chosen equally advantageous in these respects, then that which passes through the most populous and richest country in minerals ought to be chosen.

4°, It would be a great error to suppose that the line may be lengthened circuitously; because by that means, by getting easy gradients, the velocity will be much increased, since what is gained in velocity, it is obvious, may be easily lost in greater distance.

5°, It was formerly supposed (and this hypothesis has been acted upon by many engineers) that the entire line of railway should, as nearly as possible, have one uniform slope, with very good gradients, however circuitous almost the line might be made to obtain them.

6°, Now, within certain limits, this is doubtless true; but it requires great care and considerable science to be able to determine these with tolerable accuracy in practice.

7°, It has been a maxim with some engineers, that if a uniform slope is impracticable, or if it requires too great a deviation from the straight or direct line, it is necessary, at least, to endeavour to rise progressively from one extremity of the line to another, and never to ascend where it must descend again.

8°, But it is clear that such views are, within certain limits, incorrect; for, if the traction be increased by gravity, when a train or engine is impelled *up* an inclined plane, in proportion to the rate of rise, it will be diminished in nearly the same proportion when it descends, especially when the gradients are very good, never exceeding 1 foot in 300, and generally much less, in which circumstance the acceleration from gravity requires no check.

9°, On this principle, the loss of velocity in ascending one side of a rising ground or inclined plane will be nearly, but not exactly, compensated by the gain in descending the other, when the slopes are equal, and some aliquot part of it regulated by the difference, if they are unequal, and this compensation will be the more nearly equal the better the slopes are, and the more perfect our engines become. In this last case, the *ratio* of the friction on the inclined plane to that on the horizontal plane may increase, ***though the total effect will be diminished.***

10°, Hence, in tracing a line of railway, there is no inconvenience in rising higher to redescend afterwards, so long as that does not render it necessary to extend the limit of the slopes. Thus, for example, several lines uniting two given extreme points, upon which it is admitted that the same locomotive engine draws throughout the same train, will be perceptibly equal in respect to the expense of transit, whatever be the height to which they rise or to which they descend, if their lengths be equal, and if, upon any of these lines, the steepest slopes do not surpass 1 in 200, so as to produce an inconvenient or dangerous acceleration. Hence it appears that special care should be taken to diminish the length of the line of transit, to lower the limit of the slopes, and that it is unnecessary, for the sake of remarkably favourable gradients, to involve a railway company in extravagant expenses, in order to complete tunnels, make embankments, and construct viaducts, the interest of the money required for which frequently exhausting a considerable portion of the revenue of the speculation, and diminishing the dividends of the shareholders, who ought, in all cases, to receive a fair remuneration for the money they may have advanced.

11°, To select the cheapest and most efficient line of railway, depends upon the following proposition:—***To combine the distance between two given points with the gradients in such a manner as to produce the greatest effect at the least expense.***

Though this proposition, in general, cannot be solved directly, yet, by attending to the preceding principles, an approximate solution may be obtained, by the aid of the tables accompanying this work, sufficiently accurate for all ordinary purposes.

12°, In estimating the mean value of the gradients throughout a line, the value of each, with its proper sign, must be multiplied by its length, and the algebraical sum of the products divided by the length of the whole line, including the levels in the same measure, will be the mean value of the gradients, in which the signs of the ascents must be reckoned positive, and those of the descents negative.

13°, If the force of traction obtained in this way on two lines connecting the same two extreme points be inversely as their lengths; or if the product of the length of one line, multiplied by its force of traction, be equal to the product of the length of another line multiplied by its force of traction, the effects of those two lines would be equal, or equal tonnage would, by equivalent locomotive engines, be transported along each line in equal times. This follows from the fact, that, if the traction on a *unit* of the line, such, for example, as *one* mile, be multiplied by the whole length in miles, the product will be the total traction throughout the line, and it will *express* the power expended in propelling an engine throughout the whole line. Hence the relative effective powers of two lines of railway may be easily estimated, and their respective advantages and disadvantages readily determined.

14°, As the length of a line of railway is one of the elements employed to compute the expense of transit, it is clear it should be as short as convenient and sound principles will admit, because it will also reduce the time of transit. It would be committing a great error to suppose we may lengthen the line because the velocity of transport over it is great. The same principle which rendered the establishment of a railway necessary or desirable, in order to obtain a mode of transport quicker than any other, requires that the *shortest lines* should be sought after, and even to prefer them, when sometimes they appear disadvantageous in other respects.

15°, In order to ascertain the effects of slopes, experiments have been instituted to determine the amount of tractive force necessary to propel a ton of burden on a level plane or horizontal line of a well constructed railway. This, of course, varies a little with the quality of the railway, as well as with the construction of the carriages, and depends on the total amount of friction. In general it varies from 8 lb. to 9 lb. per ton, and is therefore very generally assumed at 8½ lb. per ton, an approximation, in the present state of railway carriages, not far from the truth. Now, in one ton there are 2240 lb., consequently, if 2240 lb. be divided by 8.5 lb., the quotient is 264, an abstract number, from which it is inferred that the traction on the level plane is equal to 1-264th part of the weight drawn. But, by the principles of mechanics,—*The weight moved upon an inclined plane, is to the power by which it is moved, as the length of the inclined plane is to its height.*

Suppose, for example, that a waggon enters upon an inclined plane rising 20 feet in an English mile of 5280 feet, or 1 foot in 264 feet, it follows, from the preceding analogy, that an additional 8½ lb. will be combined with that on the level, or that *twice* the force will be necessary to propel the carriage with its load *up* this ascent at the same velocity as on the level, that is, if 8½ lb. per ton be required to propel a carriage or train of waggons at the rate of 30 miles an hour on the *level*, it would require *double* that force of traction, or 17 lb. per ton, to keep up that velocity on an inclined plane or slope rising 1 foot in 264, or 20 feet in a mile.

It also follows, from the same process of reasoning, that a velocity of 30 miles an hour might be kept up on ascending that inclined plane, if the train of waggons carried a part of the load only. It is frequently observed that an undulating line, having considerably steep slopes, limits the load to what the locomotive engine can propel *up* these gradients,—a fact undoubtedly true. But no slopes so steep as to nearly stop the trains proceeding at the rate of 30 miles an hour on the level should be admitted on any railway, unless from unavoidable necessity, and in that case a stationary engine must be employed at those points where they may be required. In all lines where the gradients are not more than 1 in 300, no such occurrence can take place; and to expend large sums of money on tunnels, cuttings, embankments, and viaducts, or circuitous lines for better, must be considered a useless expenditure of the public money.

Again, if the rise be 1 in 2000, it will require an additional force of 1.12 lb. per ton, which, added to 8.5 lb., that on the level, gives 9.62 lb., the necessary tractive force *up* this inclination, similarly as before. In this way we arrive at a distinct knowledge of the exact amount of tractive power necessary to propel any load *up an inclined plane*, whatever be its rise per mile, or its inclination.

If, on the contrary, the train be moving *down* the descending plane, then the tractive force necessary on the level plane will be diminished by the effects of gravity, to keep up the same velocity on the inclined plane as on the level. Hence, if the power be constant, there will be a *retardation* in ascending the inclined plane, and a corresponding *acceleration* in descending, which will, in well-constructed railways, whose gradients do not exceed 1 in 300, nearly counterbalance each other. The modifications on this account may be obtained from the accompanying tables.

Indeed, absolute accuracy is hardly to be expected in such cases, since a sufficient number of experiments on all sorts of inclinations, in different circumstances, to be combined with mathematical investigations, have not yet been completed. In the first table, by Mr Barlow, I believe, though it may give a good approximation to the truth, it appears singular that there should be such disruption of *the law of continuity* on the descending plane at about 1 in 140. It appears somewhat strange, that a change from 1 in 140 to 1 in 160 should change abruptly the equivalent horizontal plane from 1.00 to 0.83; while from 1 in 160 to 1 in 180 it does not change at all, and even continues the same to 1 in 500, while, by the experiments of Dr Lardner, he finds a complete compensation of velocity from 1 in 177 to the dead level, and there is no dangerous acceleration on inclined planes of considerable steepness, so that, after acquiring a certain velocity, the motion becomes uniform. No doubt this must be true when the friction, combined with the resistance of the atmosphere, become equivalent to the acceleration from gravity. More numerous experiments, I suspect, are yet required to ascertain the precise limits, in given circumstances, within which this compensation takes place. Though I am disposed to put greater confidence in Mr Barlow's views on most points than in Dr Lardner's; yet, in the present case, from the remarks I have made above, and what has occurred to my own knowledge, it would appear that there is some foundation for Dr Lardner's results.

On the preceding principles will be compared the relative merits of two assumed lines of railway, in which the values of the respective gradients are given in a column adjacent to the corresponding measured distances of the slopes, &c., for a passenger train of 50 tons only, by way of example.

Results of Line A, for a Passenger-Train of 50 Tons.						
No. of Slopes	Character.	Measured Distance in Miles.	Gradients.	Equivalent Hor. Dist.		Mean Hor. Distance.
				Forward.	Backward.	
1	Level	0.800	0	0.800	0.800	0.800
2	Descent	1.530	1 in 2000	1.484	1.576	1.530
3	Ascent	2.950	1 in 2000	3.227	2.673	2.950
4	Level	1.736	0	1.736	1.736	1.736
5	Ascent	6.250	1 in 600	7.450	5.250	6.362
6	Ascent	1.143	1 in 1200	1.251	1.036	1.163
7	Level	1.143	0	1.143	1.143	1.143
8	Descent	1.143	1 in 1200	1.036	1.251	1.163
9	Ascent	2.270	1 in 2000	2.483	2.057	2.270
10	Level	0.760	0	0.760	0.760	0.760
11	Descent	2.480	1 in 1200	2.247	2.713	2.480
12	Level	0.470	0	0.470	0.470	0.470
13	Ascent	8.455	1 in 800	9.639	7.271	8.455
14	Level	2.600	0	2.600	2.600	2.600
		33.730		36.326	31.336	33.882

Hence 33.882 m. — 33.730 m. = 0.152 mile, or about $\frac{1}{7}$th of a mile, the loss upon the gradients.

Results of Line B.						
No. of Slopes	Character.	Measured Distance in Miles.	Gradients.	Equivalent Hor. Dist.		Mean Hor. Distance.
				Forward.	Backward.	
1	Level	0.875	0	0.875	0.875	0.875
2	Ascent	1.000	1 in 528	1.212	0.830	1.030
3	Level	1.500	0	1.500	1.500	1.500
4	Ascent	2.000	1 in 480	2.460	1.660	2.060
5	Ascent	4.875	1 in 422	6.138	4.046	5.115
6	Ascent	3.625	1 in 440	4.531	3.009	3.770
7	Ascent	6.625	1 in 330	8.811	5.499	7.155
8	Descent	4.000	1 in 330	3.320	5.320	4.320
9	Descent	2.500	1 in 660	2.100	2.938	2.675
10	Descent	1.759	1 in 406	1.452	2.222	1.837
		28.750		32.399	27.899	30.337

Hence 30.337 m. — 28.750 m. = 1.587, or about $1\frac{1}{2}$ mile, the loss upon the gradients.

On comparing the results of these two lines, designated by A and B, it appears that A loses only about *one-seventh* of a mile, while B loses about *a mile and a half in steam-power or in time* by means of the gradients *alone*, when the effects of the slopes are estimated by Mr Barlow's tables; but this does not give a proper estimate of the relative expenses of the lines. This is obtained from a comparison of the mean horizontal distances in the right-hand columns. Thus A has 33.882 miles of mean horizontal distance, while B has only 30.337 miles. The difference of these is 3.545 miles, the loss of A above B, in passing once along the line, and of course double of this, or about 7 miles, in one trip forward and back, of *steam-power or of time.* These conclusions are independent of 5 miles of actual measured distance, for the construction of which additional miles funds must be provided, which causes an immense loss, or useless expenditure of money to the shareholders of A's line, while that of B is more effective.

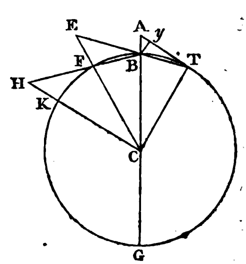
Besides entailing the expenses of construction on the shareholders for these additional useless 5 miles, the expenses of transit over them must be charged on goods and passengers, thus compelling those who use the railway to suffer a severe annual loss, without any equivalent advantage. These injudicious schemes will no longer be tolerated, as the Legislature now (1841) employ, very properly, men of competent science to examine them before an act for their execution can be obtained. So much for the benefit conferred on the public by injudicious speculators in railways. *In fact, they materially injure the public as well as themselves.*

Hence, from this investigation, great care should be taken to avoid extending the lines too much for the sake of good slopes, whereby more may be easily lost in distance than gained by good gradients. Hence, too, the policies, or fancy grounds and parks, of noblemen and private gentlemen are indiscriminately assailed without any reason. The public are, indeed, greatly interested in the proper selection of the cheapest and most economical line of railway in every respect, and ought to make every exertion to obtain it. For this purpose, it appears that a national system of railways ought to be adopted, and that parliament ought to exercise great care in examining the nature and qualities of all railways, before passing bills for their completion.

In conclusion, it ought to be an object with the engineer to render, as nearly as possible, the cuttings and embankments *equal*, so that little ground will be required for superfluous earths to be deposited in *spoil banks*, as they are technically called. For making the necessary calculations, Macniell's Tables will be found very useful. If these are considered too expensive, some of the smaller tables, as our Table XXXII., &c., may be easily obtained, accompanied by directions for their use.*

3. *To lay off points in a circle, such as in the curves of railways.* In the figure we have, by the principles of geometry, $AB = \frac{AT^2}{BG}$ nearly.

Here let the radius CB of the curve be one mile, hence $BG = 2 \times 8000$ links $= 16{,}000$ links. Now, let a point for each degree round the centre be set off. Then the natural tangent for one degree is 0.017455, whence $0.017455 \times 8000 =$ 139.64 links $= AT = Ty$ nearly, in this case. Hence $\frac{AT^2}{BG} = \frac{139.64^2}{16000} = 1.225$ links $= AB$ or yB, because, when the circle is great com-



* See Explanation of Railway Tables.

pared with TA or Ty, then AB and yB must be nearly equal.

Again, produce the chord TB to subtend another degree, making BE = TA, or rather Ty, then EF = 2yB = $2 \times 1.225 = 2.45$ nearly. Again, through B and F produce the chord BF to H, so as to subtend another degree, and make HK = 2.45 links as before, and K will be another point in the circle; continue this process till the whole curve is completed by points, and then soften the points into a continuous curve by any practical method that may readily occur.

I believe an instrument for accomplishing this has been invented by Mr Brunel, which is easy in its application and sufficiently accurate in practice.

DESCRIPTION AND USE

OF

THE INSTRUMENTS

EMPLOYED IN

TRIGONOMETRICAL SURVEYING AND LEVELLING.

DEFINITIONS

NECESSARY TO BE KNOWN IN ORDER TO UNDERSTAND THE USE OF INSTRUMENTS.

1. When angles are measured on a level plane, similar to the surface of the sea or a lake, they are called *horizontal* angles.

2. When angles are measured on a plane, perpendicular to the level plane, they are called *vertical* angles.

3. If angles are measured in neither of these planes, they are said to be taken in *oblique* or on *inclined* planes.

4. If the angles be measured in the vertical plane, above the straight line passing through the eye of the observer perpendicular to the plumb-line, they are called angles of *elevation*; their complements to 90° are called *zenith-distances*; and the angular instruments, such as theodolites, altitude and azimuth circles, &c. are commonly constructed so as to read either way according to the orders of the observer.

5. When angles are taken below the level or horizontal line defined above, they are called angles of *depression*; though, when the instrument reads zenith-distances, this dis-

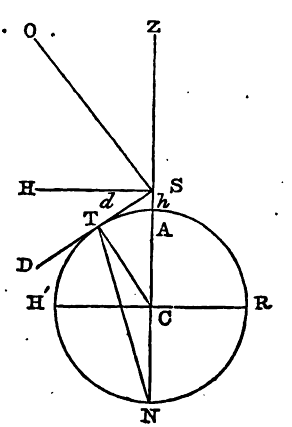
tinction is unnecessary, because the excess above 90° is the depression.

6. These respective positions are known by means of the *plumb-line* or *spirit-level*, one or other of which is generally applied to all instruments requiring, in their application, a knowledge of these planes.

7. Those points are said to be in the *same level* which are equidistant from the earth's centre considered as a *sphere*. The earth, however, is really a *spheroid*, having its polar axis less than its equatorial diameter by $\frac{1}{300}$, and in the more refined operations, it is the surface of this spheroid that is accounted the level.

8. In the measurement of altitudes, the height of the instrument must generally be added to the result from calculation when situated at the *bottom*, but subtracted when at the *top*. The mean level of the sea, or that at half-tide, is generally adopted as the standard from which heights are estimated. If high or low water at spring-tides be assumed, this should be stated, and the rise of the tide recorded.

9. To illustrate the preceding definitions and terms used in the mensuration of heights and distances on the earth's surface trigonometrically, let AH′NR be a section of the earth at the surface of the sea, considered as a sphere, which for this purpose is sufficiently near the truth, then, if S be the station of the observer at the height AS, or h, above the mean level of the sea, ZN will be pointed out by the plumb-line hanging freely, HS perpendicular to ZN will be indicated by the spirit-level;



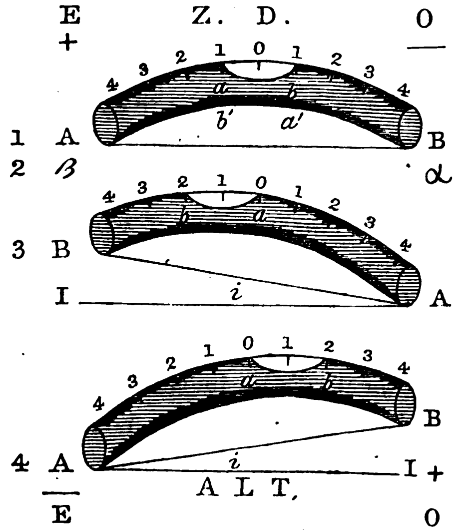
the point T will be the utmost limits of vision, or the surface of the sea at the distance TS or *d*, and to these lines distinctive names have been appropriated. The point Z is called the zenith, the opposite point N is called the nadir; HS, perpendicular to ZN, is called the horizontal line; H′CR, parallel to it, and passing through the earth's centre C, is called the true horizon; and ST the distance of the visible horizon where the sky and the extreme limits of the surface of the sea appear to meet. When observations are made with angular instruments, as the theodolite, the altitude and azimuth circles, the reflecting circle, &c. on any object O, in the direction SO; the angle OSZ is called the zenith-distance, OSH the altitude, and HST the depression of the horizon TS, below the horizontal line HS marked by the spirit-level, called also by seamen the *dip* of the horizon. Independent of refraction, it is equal to the angle TCS measured by the arc AT. This arc has by Horsburgh, &c. been improperly given as the definition of the dip, though, as has been shewn, it is equal to it, and may be taken as a measure of it only, without allowing for the effects of terrestrial refraction.

THE SPIRIT-LEVEL.

The spirit-level is a cylindrical glass-tube AOB, of a uniform diameter throughout, which is carefully ground into the form of a circular arc of large radius, occasionally of several hundred feet, which makes it appear nearly straight.

It is then nearly filled with some fluid, as alcohol or ether, and the ends are artificially closed or hermetically sealed. To the upper surface of fine instruments there is adapted a scale having divisions cut on a slip of ivory, or even on the surface of the glass itself, shewing single seconds or some multiple of the second, though in all the smaller portable instruments, *two* seconds is the best, and by far the most convenient in application, and the reading from a central zero is commonly preferred.

If the cylindrical arc be placed in a vertical plane with



the convex side uppermost, and the extremities A B resting on a horizontal surface as in the figure (1), the bubble of air *a b* left in the tube will rise to the highest part of it, and will remain, from the principles of gravity, steadily between the same divisions, while the plane on which it is placed revolves round a truly vertical axis, by that means retaining the plane in a perfectly horizontal position. If it be necessary to bring the plane of an instrument, such as that of a theodolite, readily into a horizontal position, it is generally provided with two levels nearly equal to each other in every respect, which are placed at right angles to one another, and permanently attached to the plane, though still capable of adjustment by screws for that purpose. In the more ordinary instruments, the maker marks the position of the bubbles when the plane is horizontal, and therefore when the bubbles occupy these positions, the plane on which they are fixed must be horizontal. For common instruments these marks are reckoned sufficient, and the divided scales are thought to be unnecessary.

In fine instruments, if the plane of the level be inclined, by the unequal action of heat upon its supports or other unavoidable causes, to the vertical, and the position of the extremities of the bubble be noted, then if, upon reversing the instrument by turning it half round the vertical axis, at a second observation, they occupy the same positions as in (1) (2), where A and B merely exchange places and occupy those of α and β in a reversed position, the plane will be truly level, and have the same inclination to the vertical ZN in the preceding figure as it had before. This, however, from different causes, almost never happens, and then it becomes absolutely necessary to record the reading of both ends of the level reckoned most conveniently, as in the figure, from a central zero, indicated from the positions marked (3) and (4). If the verniers of the instrument read zenith-distances, the reading of the extremities of the bubble on the scale of the level next the observer, called the *eye-end*, is marked +, and that farthest from him, or the *object-end*, –. If the instrument reads altitudes, the signs must be reversed, that is, the eye-end must be reckoned –, and the object-end +. If the divisions on the scale of the level do not shew single seconds, the difference between the positive and negative sums must be multiplied by the value of *one* division, and the result divided by *twice* the number of the observations, and applied to the degrees, &c., read from the circle, according to its sign, to give the true reading corrected for the inclination of the vertical axis.

EXAMPLE 1. Suppose the circle reads zenith-distances, then the reading of the level in the figure is marked thus :—

	e +	*o* —
No. 1 of the figure A, B gives . .	1	1
··· 2 of the figure . . .	1	1
··· 3 by a slight inclination B, A .	2	0
··· 4 by an opposite inclination A, B .	0	2
Sums . .	4	4

These sums being equal, and having opposite signs, prove that no error arises from the inclination of the vertical axis of the circle in the use of a fixed level.

Ex. 2. In a course of operations made at Broddick in Arran by the writer, with a circle having three verniers each shewing 10″, and a fixed level, each division of the scale of which indicated 3″, the following observations on Polaris were taken in latitude by estimation 55° 35′ 30″ N., longitude $20^m\ 44^s$ W., on the 6th of August 1836, by a watch $9^m\ 5^s$ fast.

Broddick Bridge, August 6. 1836.

English barometer $b=29^{in}.98$. Fahrenheit's thermometer $t=49°.5$.

Obs.	Times. h m s	Ver.	Z. D. ° ′ ″	Level. *e* +	Level. *o* —	Circle.
1.	10 23 45	A	34 17 20	20	14	Direct.
		B	17 30			
		C	17 20			
2.	10 44 45	A	34 12 20	23	11	Reversed.
Mean	10 34 15	B	12 30			
		C	12 10	Sums +43,	−25	

−25

Mean . . 34 14 51.7 Diff. +18

Effect of level . + 13.5

3″ = value of a division.

Z. D. correc. for lev. = 34 15 5.2,

2 no. obs. = 4)54

+ 13.5 = l = effect of level.

	h	m	s
To the mean of the times of observation . .	10	34	15
Apply the error of watch fast	—	9	5
Mean time at place of observation . . .	10	25	10
Longitude in time west	+	20	44
Mean time at Greenwich	10	45	54

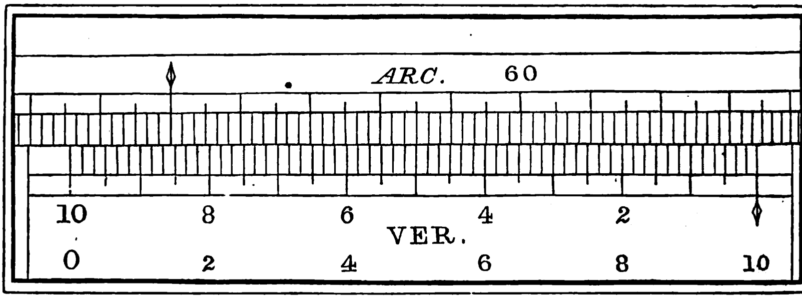
from which the latitude may be found by the method explained in the Nautical Almanac for 1836, p. 524, or by the formula given for the same purpose in this work. These observations, with the assistance of Mathematical and Astronomical Tables, and the Nautical Almanac, give the latitude 55° 35′ 28″.6 N. from this series, which, where great accuracy is required, ought to be continued for a considerable time on stars both to the north and south of the zenith, in pairs nearly equidistant from it, to destroy any error from a bias in the instrument, or a faulty habit of observing.

The mean of the whole, combined according to the number of observations in each series, will, even with a moderate-sized instrument, give the final latitude with considerable accuracy. The most convenient division of the scale of the level is 2″ for each, because the effect of the level would be got by dividing the difference of the sums of the columns *e* and *o* by the number of observations simply, whereby both the multiplication by the value of one division of the scale and the operation of doubling the number of observations for a divisor is avoided.

THE VERNIER.

The vernier is a small scale sliding against a divided scale or arc, in such a manner as to subdivide those parts of the arc into smaller divisions than can be conveniently and distinctly executed on the arc itself.

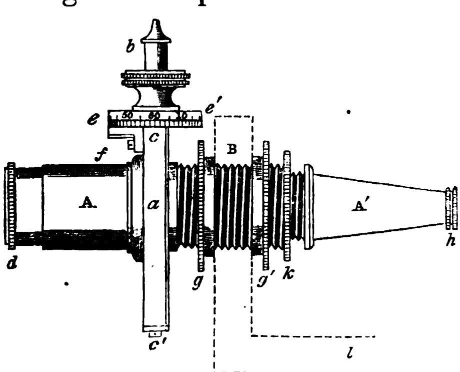
Thus, if an arc be divided into single degrees, then a small scale, having an extent equal to 59 of these degrees divided into 60 equal parts, each part on the vernier will be

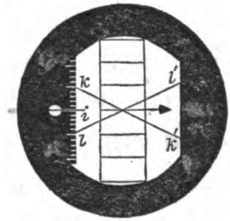


$\frac{1}{60}$ less than one on the arc. But $\frac{1}{60}$ part of one degree is equal to 1′, consequently such a vernier would, by the coincidence of any two lines—one on the vernier with one on the arc—shew single minutes. This vernier, however, would be rather inconveniently long. If, therefore, the arc, as in the common theodolite, be divided into half degrees of 30′ each, then a vernier scale of 29 of these half degrees, divided into 30 equal parts, will also shew minutes, and the vernier scale, being shorter, is more convenient. In this case, care must be taken not to forget half a degree in recording the reading indicated by the arrow which marks the degrees and parts on the arc. Generally, if $n-1$ divisions on the arc be divided into n divisions on the vernier, then this vernier shews $\frac{1}{n}$ part of the divisions on the limb, for $1-\frac{n-1}{n}=\frac{1}{n}$. Thus, if one degree, as in the figure, be divided into six equal parts of 10′ each, and if 59 of these be divided into 60 on the vernier, then $\frac{10'}{60}=\frac{1'}{6}=10''$; consequently, such a vernier shews 10″ directly, and 5″ may be easily estimated. Finer subdivisions than these are generally obtained by the reading microscope. If observations be repeated, however, on different parts of the limb, a degree of precision sufficient for almost the nicest purposes may be easily obtained even by this vernier. Indeed, repetitions should be taken on different days, to avoid the irregularities to which the most powerful instruments are liable from the effects of refraction. In using the different kinds of verniers, it will be found more easy, and less liable to error in reading off the arcs, when the degree on the limb and the minute on the vernier are similarly divided. Thus, if the limb be divided to 20′, the vernier should shew 20″; if the limb read to 10′, the vernier should read to 10″, as in the figure, &c. By this arrangement, the mind is less liable to be distracted during the operation of reading, than when the limb is read according to one arrangement, and the vernier to another.

THE READING MICROSCOPE.

When the reading microscope is applied to read the divisions of an astronomical circle, the graduations in the arc generally indicate spaces of five minutes each, which are read along with the degrees by means of an index pointer. The remaining minutes and seconds are determined by the reading microscope.





Method of Adjustment and Application to Practice.

In the figure, A, A′ represents the microscope, attached to the instrument by the arm *l*, and passing through its support B, formed by a collar embracing it, where it is firmly held by the milled nuts *g*, *g'*, acting on screws cut upon the tube of the microscope. These nuts also serve the purpose of placing the instrument at the proper distance from the divisions which it is employed to read, in order to obtain distinct vision, and destroy parallax. In the body of the instrument, at *a*, the common focus of the object and eye-glasses, are placed two wires, crossing each other diagonally at acute angles, which are made to traverse the field of view, backwards or forwards, by turning the micrometer head *b*, whose axis works in the box *c*, *c'*, in the first figure. In the second figure is shewn the field of view, with the magnified divisions on the circle, as seen through the microscope. The shaded part represents the diaphragm, with its cross wires, the angle between which may, by turning the micrometer-screw *b*, be bisected by any line on the circle within the field of view, as is shewn in the figure. On the left hand of the diaphragm appears the scale of minutes, from its shape called a *comb*, in which each tooth represents a minute. Moveable with the wires along the comb, is a small index or pointer *i*, which, in the figure, is represented at zero, the centre of the scale, known to be correct when it bisects the small hole at the back of the comb, while at the same time the cross wires bisect a division. Now one revolution of the screw *b* moves the point connected with the wires over one tooth of the comb, that is, over a space on the divided arc of the circle equal to one minute, and therefore part of a revolution moves them only over a part of a minute. To determine the value of this fractional part of a minute in seconds, a large cylindrical head, *e*, *e'*, is attached to the screw, having its exterior circumference divided into 60 equal parts, representing seconds, and read by an index opposite the eye of the observer at *f*. In reading off an angle by this instrument, observe, first, the degrees and nearest five minutes shewn by the pointer on the graduated circle, then this will be the true angle, if, as in the figure, a division on the graduated circle bisect the angles of the cross wires. But if the cross wires be not thus bisected, read the degrees and nearest five minutes as before, then apply to the microscope, and, by turning the screw *b* in the order of the numbers upon the head *e e'*, make the nearest division in the reverse order of the numbers upon the graduated circle, nicely bisect the acute angles formed by the intersection of the cross wires; the number of teeth which the pointer *i* has passed over from its zero, to produce such a bisection, will be the number of minutes to be added to the degrees and minutes read off the circle by the pointer; and, lastly, the odd seconds *and estimated tenths* to be added are taken from the divided head *e e'*, as shewn by the index *f*. In cases of great nicety, the run of the microscope may be taken to the next division in the *direct* order of the numbers upon the circle, which, subtracted from *five minutes*, ought to give the same number of minutes and seconds as formerly, to be added to the arc shewn by the pointer on the circle. If there is a slight discrepancy, the mean of both may be taken and so applied.

Adjustments of the Microscope.

1. To make the cross wires in the focus of the microscope and the divisions on the circle appear both at the same time distinct and free from parallax, draw out the eye-piece *d*, until distinct vision of the wires is obtained, and the divisions on the instrument are equally well defined and free from parallax; that is, whether any motion of the eye causes the least apparent displacement of the wires with

respect to the graduations. If such a dancing motion be observed, the microscope must be moved to or from the circle, by turning the nuts *g g'*, easing the one and tightening the other, till the wires and graduations appear both distinct, and no parallax can be detected.

2. To make five revolutions of the micrometer-screw measure a five-minute space upon the graduated circle exactly. If the run of the screw has been carefully adjusted by the maker, and no alteration made in the body of the microscope, the image of the space between two divisions will be exactly equivalent to five revolutions of the screw, when the wires and divisions are both seen distinctly. Suppose, for example, however, that the microscope has been deranged, and the run is too great, and that the 5′ space on the arc is equal to 5′ 5″, when measured by the micrometer, thus making the image too large. But the magnitude of the image formed by the object-glass of the microscope depends entirely on the distance of the object-glass from the limb, and, in the ordinary construction of the microscope, is diminished by increasing the distance between the object-glass and the limb, and conversely. In the case supposed, the image is too large, consequently the object-glass must be removed farther from the limb, by turning the screw at *h* inwards in the direction of B.* The image will not now be formed at *a*, as it ought to be, but nearer to B, and distinct vision must again be obtained by bringing the whole body of the microscope, by the screws *g g'*, nearer to the limb. By a repetition of two or three more cautious attempts in this way, five revolutions of the screw carrying the cross wires will correspond exactly with the image of the space between two divisions, which, for greater security,

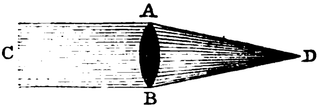
* This is sometimes done by moving the part A′, and fixing it by *k*.

may be read to the right and left on each side of zero. The screw *c′* gives motion to the comb or scale of minutes; and the micrometer-head, being adjustable by friction, can be made to read either zero or any required second, when the cross wires bisect any particular division, by holding fast the milled head *b*, and at the same time turning the divided arc *e e′* round till any required division, as zero, coincides with *f*, the index.

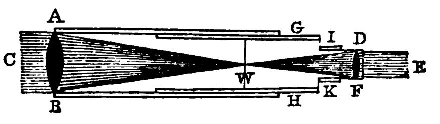
THE TELESCOPE.

All instruments capable of now giving results possessing the requisite accuracy are furnished with one or more telescopes. The rays of light proceeding from distant objects move in straight lines, unless they are reflected or refracted by some *medium*, such as metal, glass, &c., and also in *parallel lines* nearly, especially if the object from which they come be remote.

Let AB represent the section of a lens, such as the object-glass of a telescope. Let the parallel rays coming from some distant object on the left beyond C strike the glass lens



AB, they will pass through it, suffering refraction, and, on leaving the lens at the opposite side, they will converge and meet in the straight line CD at a certain point D, called the focus of the lens, where the eye, by a little practice in selecting the proper distance, will see an inverted image of the distant object in the air. Now, suppose two of these lenses are applied to the construction of a telescope, the image of a distant object will be



formed at W, the focus of the object-glass AB, where, by moving the eye-glass or lens DF till its focus comes to the same point W, by means of two slides GH, IK, the eye of the observer at E will view a magnified inverted image of the object formed by the object-glass AB, with the eye-glass DF as a microscope. Since both these lenses are capable of motion, they may always be moved in such a manner that their foci will meet exactly at W, making the central line CWE a straight line, technically called the optical axis, or line of collimation of the telescope, from which the readings in all mathematical and astronomical instruments are taken. This point, W, is marked by fine wires, hairs, silk fibres, or spider lines, and this is the reason why both glasses must be moved till the telescope produce distinct vision, and the wires are well seen, in which case the telescope is said to have no parallax.

If this adjustment is imperfect, the object will, on moving the eye up and down a little, start from the intersection of the wires, thus causing an uncertainty in all observations, which must be instantly corrected. The point W, or focus of the object-glass, varies with every change in the distance of the object, and therefore this adjustment must be frequently examined, and, if necessary, corrected for terrestrial objects, though it remains constant for celestial. This instrument is commonly called the astronomical or inverting telescope, because it wants other two lenses between the object and eye glasses to view objects erect as they appear to the naked eye. They are, however, almost universally employed for astronomical purposes, where it is less necessary to see objects erect, and because they appear more distinct, from a greater quantity of light being attainable, since each lens absorbs a portion of it. It is scarcely necessary to add, that no attempt at adjustment should be made *during*, but always *before*, an observation.

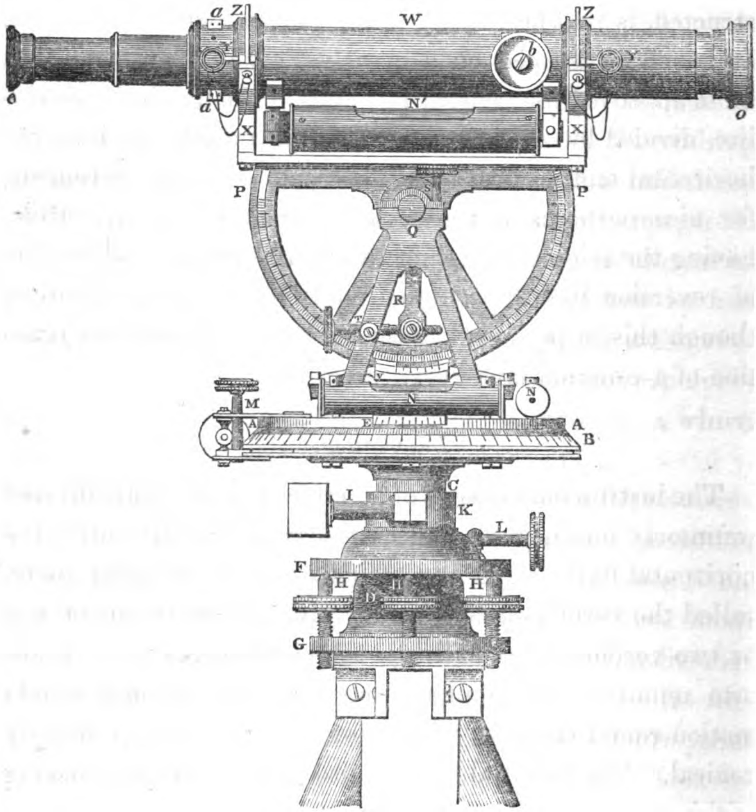
THE THEODOLITE.

Of all angular instruments, the theodolite, properly constructed, is that best suited to the purposes of the surveyor. It has been formed on a great variety of plans, but that most approved for general purposes is the five-inch theodolite, divided by one or two verniers to minutes in both the horizontal and vertical arcs. It would be an improvement, for nice purposes, if the vertical arc were an entire circle, having the telescope passing through its centre, and capable of reversion like the common portable astronomical circle, though this is perhaps unnecessary for the ordinary practice of a common land-surveyor.

Description.

The instrument consists of an under circular plate divided commonly into degrees and half-degrees, usually called the horizontal limb at AA, on which an upper circular plate, called the vernier-plate, turns freely, that by means of one or two verniers, E, subdivides the half-degrees on the lower into minutes. Both plates have an easy though steady motion round the axis, which, for that purpose, is slightly conical. The internal centre also fits into a ball working within a socket at D, and the parts are held together by an internal screw at the lower end of the axis, within the tripod formed by the legs.

The diameter of the lower plate is a little greater than that of the upper, and its exterior edge is cut off in a plane inclined to the axis, which is technically called chamfered, and in the best instruments is covered with *silver*, to receive the graduations, being less liable to become obscure by the action of the atmosphere than the metal of which the plates are made. On the opposite ends of an imaginary diameter, at the distance of 180° from each other, a small space, E, is also chamfered and covered with silver, form-



ing, with the edge of the lower plate, a continued inclined plane, on which the proper divisions being cut constitute the verniers. When the lower limb is graduated to thirty minutes, the vernier has a space equal to twenty-nine of them divided into thirty, and each, consequently, reads to minutes, which, by means of a microscope, either attached or detached, may by estimation, when thought necessary, be carried to thirty or even twenty seconds. For fine purposes, the degree is divided into three equal parts, each of twenty minutes, and, on the vernier-plate, a space equal to fifty-nine of these being divided into sixty, then this vernier indicates twenty seconds.

The two parallel plates under the graduated limb at F and G are held together by a ball and socket at D, and are set firm by four milled-headed screws H, H, H, H, which turn in sockets fixed to the lower plate, while their heads press against the under side of the upper, thus, acting on the vertical axis by means of the ball and socket, render the horizontal and vernier plates truly level when the instrument is prepared for observation.

Beneath these parallel plates is a female screw, within which the male screw lays hold of the axis, and keeps it firmly to the stand. The lower parallel plate is connected by brass joints to three mahogany legs, having their lower ends pointed with metal, for entering the ground, and frequently so constructed, that, when shut up, they form one round staff, secured in that form for carriage by rings placed upon them. When the legs are opened out they make a firm stand, however uneven the ground may be. Sometimes the legs are round or cylindrical, and formed of two parts, which unscrew for packing in a box, to facilitate their carriage when travelling. In this case, shods should be prepared to screw on the upper half, to be used when any nice observations are to be made requiring great steadiness in the stand.

The lower horizontal limb can be fixed in any given position by the clamping-screw I, which causes the collar K to embrace the axis C, and prevent it moving. It is generally necessary that the telescope should be fixed in some precise position, more exactly than it can be by the hand alone. For this purpose, it is first made nearly correct by the hand, the parallel plates being previously clamped with the verniers, and by the tangent-screw accurately set to the given positions, as zero and 180°; then the instrument is moved a small quantity, by turning the slow-motion screw L, attached to the upper parallel plate, till the direction of the cross-wires of the telescope is perfected. In a similar manner, the upper or vernier plate being now released, the telescope may again be placed upon any other object whose angular distance from the first is required, which, by the clamping and tangent screws, may be rendered perfect as before, and the angle shewn by the verniers must now be read and recorded. Before proceeding to measure the horizontal and vertical angles, the parallel plates carrying the divisions and verniers must be made perfectly horizontal by two spirit-levels N N, placed at right angles to each other, and rectified by their adjusting screws for this purpose. Upon the vernier-plate, too, is commonly placed a compass between the levels, for the purpose of taking magnetic bearings.

The vertical frames Q Q support the pivots of the horizontal axis of the vertical arc PVP, on which the telescope is placed. There is sometimes an arm carrying a microscope for reading the altitudes and depressions measured by this arc, and determined by the vernier V, which has a motion of several degrees, so as to be placed opposite the divisions of coincidence. There are, on this vernier, two sets of divisions reading in opposite directions, of which the upper reads elevations, and the lower depressions.

Another screw S clamps the end of the horizontal axis seen at Q, while a slow-motion or tangent-screw, T, moves the vertical arc and telescope, till a perfect observation be made. One side of the vertical arc is inlaid with silver, and is divided into single minutes, or lower, with the assistance of its vernier. On the other side there are sometimes placed divisions, to shew the difference between the hypotenuse and base of a right-angled triangle, the hypotenuse being 100, or, which comes to the same thing, the number of links to be deducted from each chain's length in measuring up or down an inclined plane, to reduce it to the horizontal measure. If the angle of elevation and depression be taken, these afford data to take this reduction more accurately, from a table calculated expressly for this purpose, or the deduction may be readily made by a table of natural versed sines. The level which is shewn at N′, under and parallel to the telescope, is attached to it at one end by a joint, and at the other by a capstan-headed screw, and will permit the level to be placed parallel to the optical axis of the telescope, commonly called the line of *collimation*. The screw at the opposite end is employed to adjust it laterally, so that it may be placed parallel to the axis also in a vertical plane. In this way the level is placed parallel to the axis of vision, both horizontally and vertically. The telescope has two collars or rings of gun-metal, ground truly cylindrical, on which it rests on its supports XX, called Ys, from their resemblance to that letter, and it is confined in its place by the clips Z Z, which may be opened by removing the pins Y Y, for the purpose of reversing the telescope in double observations, when great accuracy is required. These pins should, to prevent loss, be secured by silk strings connecting them with the frame.

In the focus of the eye-glass are frequently placed three fine wires or lines of spider-web, one horizontal and two crossing it nearly vertically, making with each other a small or acute angle. This method of fixing the wires is preferable to having one horizontal and another vertical, crossing one another at right angles, as is commonly the case, especially for horizontal angles, because a distant object can be made to bisect the small angle between the vertical wires with more certainty than the object can be bisected by the vertical wire. For many astronomical purposes, however, the second method is preferable; and for making observations on the sun, one or two coloured glasses should be provided, to be fitted on the eye-end of the telescope. The screws for adjusting the cross-wires are shewn near the eye-end of the telescope at *a*, *a*, *a*, *a*, of which there are four, equidistant from each other. Hence the imaginary line joining any two opposite screws is at right angles to the line joining the other two, so that, by *first* easing the one, and *then* tightening the other opposite to it, the intersection of the cross-wires may be readily adjusted. The object-glass, *o*, is moved by turning the milled-head *b*, on the side of the telescope, till the object is seen well defined, while a corresponding motion is given to the eye-glass, *e*, by moving it with the hand in its slide till the wires are seen equally distinct, which will easily be effected in one or two trials. The reason and effects of this process will be readily comprehended by consulting the description of the last figure, p. 99, though the arrangement there is somewhat different.

A brass plummet and line are also packed in the box with the theodolite, to be suspended from a hook truly under the centre of motion of the horizontal arc, by which it can be placed exactly over the station whence the observations are taken, an operation to be carefully performed in all fine work, otherwise considerable errors may arise, and surveys cannot close accurately. If required, two extra eye-pieces are furnished for the telescope, to be used in astronomical observations. The one inverts the object, has a greater magnifying power, but, with fewer lenses, possesses more light. The other is a diagonal eye-piece, which, without inconvenience, will enable an observer to see objects having a considerable altitude. A small cap, containing a dark-coloured glass, is made to apply to the eye-end of the telescope, or to either of the preceding lenses, to screen the eye of the observer from the effects of the sun's rays, when that object is observed. A magnifying-glass, a screw-driver, and a steel-pin to turn the capstan-screws for adjustments, are also furnished with the instrument. In some theodolites, the telescope passes through the horizontal axis, the supports are made sufficiently high to allow the telescope to pass under them when the instrument is reversed in azimuth, and it then becomes an astronomical altitude and azimuth circle. With these additions, a well-made theodolite may perform most of the problems in practical astronomy with considerable accuracy, though such an instrument would be rather too good for the usual purposes of surveying, which may be very well effected by an inferior instrument.

Adjustments.

1. The first adjustment is to make the intersection of the cross-wires coincide with the axis of the cylindrical rings on which the telescope turns, called rectifying the line of *collimation.* This is known to be correct when the eye, looking through the telescope, observes the intersection of the wires continue on the same point of a well-defined distant object during an entire revolution of the tube of the telescope in the Ys. First make the intersection of the wires, when the level is under the telescope, coincide with some well-defined distant object; then turn the telescope half round in its Ys till the level lies above it; and if the same point is still cut by the intersection of the wires, the adjustment is correct in that position. If not, move the wire one-half the deviation, by turning two of the opposite screws at

a a, taking care to *release one* before tightening the other, and correct the other half by elevating or depressing the telescope. Proceed in like manner with the other position, by placing the level alternately on the right and left.

Now if the coincidence of the cross wires with the mark remains exact during a complete revolution of the telescope in the Ys, the line of collimation is correct; if not, the same operations must be repeated till it is so.

2. The second adjustment is that which places the level attached to the telescope parallel to the *rectified* line of collimation. The clips Z Z, being open, and the vertical arc PVP clamped, bring the air-bubble, N′, of the level to the centre of its glass-tube by turning the tangent-screw T; when this is done, reverse the telescope in the Ys, that is, turn it end for end very carefully, so as not to disturb the vertical arc; then, if the bubble resume its former position in the middle of the tube, all is right; but if it rises to one end, bring it back one-half by the screw towards the eye-end of the telescope in the figure, which elevates or depresses that end of the level, and the other half by the tangent-screw T; and this process must be repeated till the adjustment is perfect. To make it completely so the level should be adjusted laterally, that the bubble may remain in the middle of the tube when slightly inclined to either side of its usual position directly under the telescope. This is effected by giving the level such an inclination, and if the bubble does not continue still in the middle, it is necessary to make it do so by turning the two lateral screws in the end of the level next the eye. If, in making the lateral adjustment, the former should be deranged, the whole operation must be carefully repeated.

3. The third adjustment is that which makes the axis of the horizontal limb, or the azimuthal axis, truly vertical.

Set the instrument, by the eye, as nearly level as possible; fasten the centre of the lower horizontal limb by tightening the staff-head by the clamp I, while the upper limb is at liberty to be moved till the telescope is over two of the parallel plate-screws; when in this position, bring the bubble of the level under the telescope to the middle of its tube by the screw T; now turn the upper limb, or vernier-plate, half round, that is, through 180° from its former position, then, if the bubble returns to the middle of its tube, the limb is horizontal in that direction; but if not, half the difference must be corrected by the parallel plate-screws over which the telescope lies, and the other half by elevating or depressing the telescope from turning the tangent-screw T, of the vertical arc. When this is effected, turn the upper limb 90° from its present position, either forward or back, that the telescope may lie over the other two parallel plate-screws, and from their motion set it horizontal by means of its level. Having now levelled the limb-plates by means of the telescope's level, which is commonly the most sensible upon the instrument, the air-bubbles of the levels fixed upon the vernier-plate may be brought to the middle of their tubes by the screws which fasten them to their places.

4. The fourth adjustment is that which brings the zero of the vernier of the vertical arc to zero on the limb. When all the preceding adjustments are perfect, if zero on the vernier does not coincide with zero on the arc, the deviation must be rectified by releasing the screws by which the vernier is held, and then tightening them after having made the proper adjustment. As this is an operation difficult to be performed accurately, it will be perhaps better to call the quantity of deviation an *index error*, to be applied according to its sign, which must be carefully noted. This index-error is best determined by repeating the observation

of an altitude, or depression in reversed positions of the telescope and vernier-plate, then half the difference will be the error; or half the sum of the observed altitudes or depressions before and after reversing the telescope, will be the true angle independent of index-error.

The Method of observing with the Theodolite.

The instrument being placed, by means of its plumb-line, exactly over the station whence the angles are to be taken, and set level by the parallel plate-screws, then, by the clamping and tangent screws, set the vernier A exactly to zero, and B to 180°, or as near it as the construction of the instrument will permit, read off the verniers, and note them in a book for that purpose. Turn the telescope by hand till it is nearly on the left hand object by a motion of the head of the instrument fixed in one piece, round the lower axis C, tighten the clamping-screw I, and with the tangent-screw L, make the intersection of the wires nicely bisect the object. Now release the upper plate, and move it round by hand till the telescope is directed to the second object, whose angular distance from the first is required; then clamping it with the screw M, make, with its tangent-screw, adjacent to it, the cross wires bisect this object correctly, and read off the two verniers as before, the difference between the first and second means will be the true horizontal angle required; thus,

First reading.		° ′ ″	Second reading.		° ′ ″
Vernier A	.	0 0 0	Vernier A	.	61 45 30
B	.	0 0 30	B	.	46 20
1st sum	.	0 0 30	2d sum	.	11 50
1st half	.	0 0 15	2d half	.	61 45 55
			1st half	.	0 0 15
			Difference or true angle	. .	61 45 50

By means of the motion of the lower horizontal plate about the vertical axis, any angle required with great accuracy may have its value repeated as often as we please, and the amount of the whole, divided by the number of repetitions, will give the simple value, almost independent of errors of construction and dividing. To repeat an angle, therefore, after making the second bisection as directed above, let the upper plate remain clamped to the lower, while the clamp of the axis is released. Now move the whole head of the instrument by hand round upon the lower axis toward the first object, placing the cross wires in contact with it, and make, as at first, the bisection perfect with the lower tangent-screw. Leaving the instrument in this position, release the upper or vernier plate, turn the telescope towards the second object as formerly, and bisect it nicely with the aid of the upper clamping and tangent screws. This operation completes one repetition, and when the observation is read off and compared with the preparatory reading of the verniers, the difference will be twice the real angle.

	°	′	″
Vernier A	123	31	30
B		32	0
Half the sum or mean	123	31	45
First reading . . . . . . —	0	0	15
Difference equal to twice the arc . .	123	31	30
Half or simple value	61	45	45

The correct angle from one repetition, and this process may be carried as far as five or ten times, if thought necessary, taking always care to read the first observation, and record it, so that when the last division is performed, as many circumferences of 360° may be first added as will render the quotient nearly the same, within a few seconds, as the first observation already recorded. When the art of constructing and dividing instruments was less perfect than at present, considerable advantage resulted from this repetition, though now little, in ordinary cases, will be obtained.

The magnetic bearing of an object is taken by reading the angle pointed out by the compass-needle when the object is bisected by the telescope, recollecting that the north end of that needle is indicated by a notch or small brass pin passing through it horizontally, and in the usual construction of the instrument, the *south* end is generally that read, though, for greater accuracy, the mean of both may be taken.

The bearing may be obtained a little more accurately by clamping the lower plate, then by moving the upper plate till the needle reads zero, at the same time reading the horizontal limb; now, by turning the upper plate about, bisect the object and read again, the difference of these two readings will be the bearing required.

In determining the variation by the theodolite-compass, it would contribute to accuracy by destroying the errors of centering of the needle, to observe two objects whose azimuths had been accurately found astronomically both *forward* and *backward*.

In taking angles of elevation or depression, it may be added that the object must be bisected by the horizontal wire, or more accurately by the intersection of the wires. In cases requiring great accuracy, after an observation is made with the telescope in its usual position, it may be reversed in the Ys, that is, turned end for end, and the same observations repeated, and a mean of the whole taken for the true value.

The proof of the accuracy of a number of horizontal angles taken completely round one point or station, is that their sum should be exactly 360°.

If all the angles of a plane triangle be measured, their sum ought to be 180°; of those of a four-sided figure 360°; and, in general, when all the angles of any polygon of n sides are measured the sum $s = 180° (n-2)$, provided all the angles be *salient*, that is, projecting outwards from the body of the figure, or if the interior angle, when even greater than two right angles, be thus measured.

THE SPIRIT-LEVEL.

The spirit-level, as usually constructed, is an instrument in some respects similar to the theodolite, and by the latter the operations of the former may be readily performed. The spirit-level has a stand with clamping and tangent-screw, a telescope with its level, and a compass exactly similar to the theodolite, but without horizontal or vertical arcs, the compass alone being thought sufficient for every angular purpose required in the use of this instrument.

The method of setting up and adjusting for observation the Y level at least, being so similar to that followed for the theodolite, that it is not necessary to say much in regard to it here. There are, however, several other kinds of levels, such as Troughton's, Gravatt's, with more powerful telescopes than those generally applied to theodolites, in which some of the adjustments are effected by the maker, and do not so easily get out of order as those of the common Y level. These adjustments are generally made in the field by interchanging the position of the instrument and divided levelling-rod, half the difference of the reading is the correction of the level, which must be corrected by altering the adjusting screws of the telescope, till the intersection of the cross wires cuts the middle point between the two readings in both positions.

There are various levelling-rods constructed, to be used along with this instrument, having marks or vanes that slide up and down, and are moved by the bearer or assistant. It is, however, more convenient, and less liable to error, to have a rod divided into feet, tenths, and hundredths, and so distinctly marked that the principal observer may easily read them through his telescope at a moderate distance, and instantly record them. *In all observations the reading and writing should be re-examined to see that both are correct.*

THE ALTITUDE AND AZIMUTH CIRCLE.

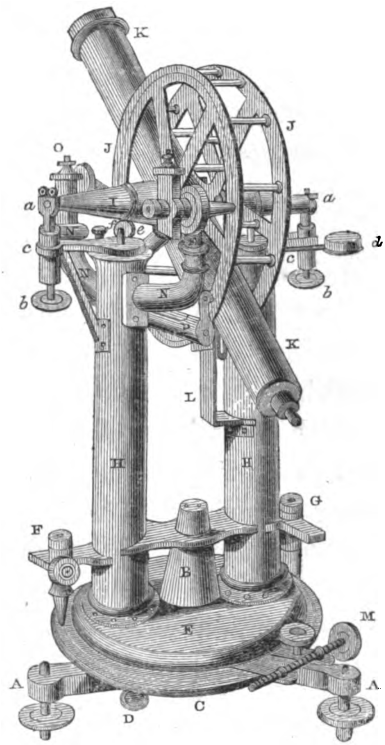
The Altitude and Azimuth Circle, as now constructed, is an instrument of great utility and importance in Practical Astronomy and Geodesy. It is made of all dimensions, varying from the small portable instrument, whose divided circles are five or six inches in diameter, to those of two or three feet. The smaller class have their arcs read by verniers, the larger by reading microscopes. The diameters of the divided circles of that whose figure is here given, are generally about twelve or eighteen inches, and the divisions are subdivided by reading-microscopes; while the smaller class of the same construction, varying from ten to twelve inches, have only verniers reading to about 10″; but both these are occasionally varied to suit the views of purchasers, and the work they are required to perform.

Description.

To the centre of the tripod A A, is fixed the vertical axis of the instrument of a length equal to about the radius of the circle passing up the interior of frustum of the cone B.* On the lower part of the axis, in close contact with the tripod, is centered the azimuthal circle C, which,

* This axis is equally or more conveniently situated, when it descends through the tripod.

by means of a slow-motion screw, whose milled head appears at D, admits of a horizontal circular motion of some extent for the purpose of bringing its zero exactly into the



meridian; though in some instruments, for the sake of permanent stability, this is omitted, as it is occasionally purchasing a convenience at the risk of some error.

Above the azimuth circle, and concentric with it, is placed a strong circular plate E, which carries the whole of the upper works, and also a pointer to shew the degree and nearest five minutes on the azimuth circle, while the remaining minutes and seconds are obtained by the reading-microscopes F G, as previously explained in the description of the reading-microscope. This plate, by means of the conical part B, supported by a brace and carefully fitted, rests on the axis of the tripod, and moves concentric with it. The conical pillars H H, support the horizontal or transit axis I, which being longer than the distance between the centres of the pillars, requires the projecting pieces *c c*, fixed to their tops to carry out the Y's *a a*, to the proper distance for the reception of the pivots of the axis. The Ys are capable of being raised or lowered for levelling, &c. the axis by means of the milled-headed screws *b b*. The weight of the axis, with the load that it carries, is prevented from pressing so heavily on its bearings as to injure the pivots by two friction-rollers on which it rests, whereof one of them is shewn at *e*.

This is accomplished by a spiral-spring fixed in the body of each pillar, which presses the rollers upwards with a force nearly equal to the superincumbent weight. These rollers, on receiving the axis, yield to the pressure, and allow the pivots to find their proper bearings in the Ys, while, at the same time, they relieve them from a great part of the weight which might cause them to wear rapidly and irregularly, thereby injuring the accuracy of the instrument.

The telescope K K passes through the axis I, on which, as a centre, there are fixed the two circles J J, each close against the telescope on both sides. The circles are fastened together by small brass pillars, and, in the larger classes of instruments, occasionally supported by diagonal braces. By this double circle the vertical angles are measured on graduations cut upon a ring of silver, generally on one of the sides only, which from that circumstance is called the face of the instrument, a distinction to be attended to in making observations, by placing it alternately to the right and left, when a series is being completed. The clamp for fixing, and the tangent-screw for giving slow motion to the vertical circle, are placed beneath it, between the pillars H H, and attached to them as seen at L. A similar contrivance for regulating the azimuth circle, likewise divided on silver, is represented at M. The reading-microscopes for the vertical circles are carried by two arms N N, bent upwards near their extremities, and attached towards the top of one of the pillars, one of which is shewn above *e*, and the other under *o*.

A circular plate of brass, with a round hole cut in it called a *diaphragm*, is fixed in the principal focus of the telescope near the eye-end, across which are stretched five vertical and five horizontal wires, at right angles with each other. The intersection of the two central ones, denoting the optical axis of the telescope, is the point by which an object ought to be bisected when only observed at one point, such as a terrestrial object when taking angles for geodesical purposes. The vertical wires are used for the same purpose as those in the transit instrument for observing the passage of a celestial body over the meridian, and the horizontal ones for taking zenith-distances or altitudes of celestial objects, by which a mean of five observations, or rather contacts, may be readily obtained. A micrometer, having a moveable wire, is sometimes attached to the eye-end of the telescope of the larger instruments, though it is not generally applied to the smaller class. This is frequently useful, but it cannot in general be so confidently relied on as an observation taken in the usual manner. The illumination of the wires necessary at night, is effected by a lamp, supported near the top of one of the pillars as at *d*, and placed opposite the end of one of the pivots of the horizontal axis, which being perforated, admits the rays of light to the centre of the tube of the telescope, where, falling on a perforated diagonal reflector, they are thrown towards the eye, and illuminate the field of view so as to enable an observer to bisect a star at or close upon the intersection of the central wires.

The vertical circle is usually divided into four quadrants, especially if there be two microscopes or verniers only, each numbered from the horizontal points, when the telescope is in a vertical position, 0°, 10°, 20°, &c. as far as 90°. In this case each microscope shews zenith-distances. If verniers are used, there must be two setts of numbers on them reading in opposite directions, as shewn on the figure, page 94. In reading observations, the *arrow* always indicates the degrees and every ten minutes; but if, when the *face* of the circle is on the *right*, the minutes and seconds obtained by the vernier be read *from* the arrow in the order of the upper numbers upon the vernier plate, then when the circle is reversed, thereby placing the *face* on the *left*, they must be read *towards* it in the order of the lower numbers, and *vice versa;* while care must be taken by the observer *always* to read the arc and verniers in the order of the figures in the *same* direction.

In some circles a different plan is followed. The whole vertical circle is divided into four quadrants as before, each numbered 0°, 10°, 20°, &c. as far as 90°; but instead of the previous method, following one another in the same order of succession. Consequently, in one position of the instrument, *altitudes* are read off, but with the face of the instrument reversed, *zenith-distances;* and with such instruments an observation is not considered complete till the object has been observed in both positions. In the latter case, the sum of the two readings will always make 90°, provided there be no error in the adjustments, the circle, or the observation. In cases where there are three or more microscopes, the readings will be different, according to the construction. When, however, there are but two microscopes O O, the straight line joining them should pass through the horizontal diameter of the circle, to render which perfect, a vertical motion, by means of the screws *b b*, is given to the Ys, to raise or depress them till this adjustment is accomplished.

A good spirit-level P, suspended from the arms which carry the microscopes, shews, upon turning round the circle, when the vertical axis is set perpendicular to the horizon. A scale usually shewing either single seconds, or (what is more convenient for small instruments) *two seconds*, is placed along the glass-tube of the level, which exhibits either the permanency, or the inclination of the vertical axis. This should be examined repeatedly, whilst making a series of observations, to ascertain whether any change has taken place in the position of the instrument after its adjustments have been completed, and, by recording its indications, to allow for any deviation if necessary. One of the points of suspension of the level is moveable by means of a screw *f*, for the purpose of adjusting the bubble. A riding-level, similar to that employed to level the transit-instrument, rests upon the pivots of the axis. It ought to be carefully passed between the radial bars of the vertical circle when set up in its place, and must be removed as soon as the operation for levelling the horizontal axis is performed.

The whole instrument is supported upon three foot-screws placed at the extremities of the three branches which form the tripod, and brass cups are placed under the ends of foot-screws when put upon its stand. A stone pedestal, set perfectly steady, is the best support for this as well as the transit-instrument; but for travellers, a strong well-made tripod of wood, firmly braced, will be the most convenient. The author has frequently used a very convenient small sixinch circle, differing a little from this, having *three* verniers, each shewing 10″, and a *fixed* level, each of whose divisions indicate 2″.

The Adjustments.

1. *To make the vertical axis perpendicular to the horizontal plane.*

Set up the instrument in the position where the observations are to be made, then turn the instrument round till the spirit-level P is lengthwise in the direction of two of the feet-screws, when, by their motion, the air-bubble in the level must be made to occupy the middle of the glass-tube shewn by the divisions of the scale attached to the level. When this is done, turn the instrument half round in azimuth; and if the axis is truly vertical, the bubble will again settle in the middle of the tube, but if not, the amount of the deviation will show *double* the quantity which the axis deviates from the vertical in the direction of the level. This error must be corrected—one-half by two of the feet-screws over which the level is placed, and the other half by raising or lowering one end of the spirit-level itself by the screw represented at *f*. This process of reversion and levelling should be repeated to ascertain whether the adjustment has been accurately performed or not, since adjustments of every kind can be made perfect by successive approximations only. When this part of the adjustment is satisfactory, turn the instrument round in azimuth a quarter of a circle, so that the level P may be at right angles to its former position; and it will then be over the third foot-screw, which must be turned till the air-bubble is again central, and this adjustment will be completed. If the whole has been correctly performed, the air-bubble will remain steadily in the middle of the level, indicated by the divisions of its scale, during an entire revolution of the instrument in azimuth. If not, the operations must be repeated till it does so.

2. *To set the vertical circle at such a height that its two reading-microscopes shall be directed to two opposite points or zeros in its horizontal diameter.*

This is readily accomplished by raising or depressing the Ys by means of the screws *b b*, which carry the horizontal axis.

3. *To level the horizontal axis.*

This operation is performed by means of a riding-level. Apply this level to the pivots, bring the air-bubble to the middle of the glass-tube by observing if the extremities of the bubble stands opposite the same division on each end of the scale by means of the screws *b b*, as before. Then reverse the level by turning it end for end; and if the air-bubble still, as formerly, remain central, the axis will be horizontal; but if not, half the deviation must be corrected by the screws *b b*, and the screw at one end of the level which raises or depresses the glass-tube of the level with respect to its supports that rest upon the pivots. After performing this adjustment, the preceding must be examined to see if it be deranged by the last process. Indeed, it is preferable to set the axis horizontal first, and then by equally raising or depressing the two ends, to bring the microscopes into a diameter, and finally to level the axis again.

4. *To adjust the line of collimation.*

This adjustment requires the middle vertical wire to describe a great circle, and the middle horizontal wire to have a certain definite position with respect to the divisions on the limb. It is usual to rectify the middle vertical wire first, the others being set parallel by the maker. Direct the telescope to some small well-defined distant object, bisect it with the intersection of the two central wires, and clamp the circle in that position. Now, turn the whole instrument half round in azimuth *exactly*, and, by the tangent-screw, elevate or depress the telescope, till it cut the same object, and if it be bisected at the same point as before, the collimation adjustment is correct; if not, turn the small screws which hold the diaphragm near the eye-end of the telescope through one-half of the error, and the adjustment will be completed. But as half the deviation may not be correctly estimated in moving the wires, it is necessary to verify the adjustment by moving the telescope the other half. This operation must be repeated till, by continued approximations, the adjustment is found to be perfect. To adjust the middle horizontal wire, point the telescope to a very distant object, near the intersection of the wires, bisect it by the middle horizontal wire, and read off by the microscopes the apparent zenith-distance. Now, reverse the instrument in azimuth, and, turning the telescope again upon the same object, bisect it as before, then read the arc which they shew. One of these, in this construction of the instrument, will be an altitude, and the other a zenith-distance; and, if there be no error, the sum of the two readings will be 90° exactly. If they do not make 90°, *half* the difference from 90° will be the error of collimation. If the instrument shews zenith-distances only, then half the *difference* of the arcs in opposite positions will be the index or collimation error, and its sign must be marked, whether + or −, when the face of the circle is to the right or left. This error may be either employed to correct an observation made with the instrument during its continuance in one position, or removed in the following manner. Read the zenith-distances in opposite positions of the circle, that is, with the face alternately to the right and left, of which take the mean, that will be the *true* zenith distance. Then, while the telescope bisects the object, the microscopes, by their proper screws, must be adjusted so as to read that mean. In making a series of observations, however, they are generally taken in pairs, with the face of the circle alternately to the right and left, consequently the mean of the readings gives the true zenith-distance, independent of the error of collimation,—a method commonly followed in practice.

5. *To set the central or middle wire truly vertical.*

This may be effected by directing the telescope to a well-defined distant object. If, on elevating and depressing the telescope, it is bisected by every part of the wire, that wire must be truly vertical. If not, it should be adjusted by turning the inner tube, carrying the diaphragm or wire-plate, till the preceding test of its verticality be satisfied; and, to avoid the effects of any small error on this account, care must be taken to make important observations near the centre only. The other vertical wires are, by the maker, placed equidistant from the middle one, and parallel to it, so that, when it is adjusted, the others are likewise correct. He also places the transverse wires at right angles to the middle vertical wire. These adjustments are always performed by the maker, and are little liable to derangement.

In general, it may be remarked, that during a series of observations, should the instrument be found to be a small quantity out of level (the other adjustments being perfect), it may be restored generally by means of the foot-screws only, when they require but a slight touch to effect it. This is more especially essential when the level of the horizontal axis is the one deranged, since correcting it by moving the Ys would derange the adjustment of the vertical circle with regard to its reading microscopes,—an occurrence which must be carefully avoided. The error of the vertical axis is to be detected by the hanging level, and, by reading its scale, can be very readily allowed for in computing observations, as has already been shewn in the description of the level.

General Rule. ***When great accuracy is required, it is both easier and safer to correct by calculation, than to adjust by mechanical contrivance.***

Use of the Altitude and Azimuth Circle.

This is the most generally useful of all instruments, because it measures with great accuracy both horizontal and vertical angles. It does not, however, possess the power of repetition, like the circle of Borda, but the effect of any error of division on the horizontal circle may be diminished or destroyed by measuring the same angle upon different parts of the arc. For this purpose, let r be the number of repetitions required, v the number of verniers, and c the change of zero in degrees, $c = \frac{360^\circ}{vr}$. Let, for example, $v = 3$, and $r = 4$, then $\frac{360^\circ}{12} = 30^\circ$, the change. Whence the successive zeros, or rather starting points, of the vernier A are 0°, 30°, 60°, and 90°. By this means the whole circumference of the circle is equally employed, by which means the small errors of excess and defect mutually destroy each other. Even a *small* quantity of change, by means of the screw D, if a great one be inconvenient, will greatly diminish the chance of errors in division, reading, and pointing. A repeating-stand is frequently added to this instrument, which is a convenient appendage when great accuracy is required in the measurement of horizontal angles; and the operation is exactly similar to that explained when treating of the use of the theodolite. The vertical angles should, in all practicable cases, be taken at least twice, reversing the circle before taking the second observation, which will eliminate not only the errors of centering and division, but also those of collimation. In applying the instrument to astronomical purposes, this method is always employed. When the instrument is used to determine the latitude by what is termed *circummeridian* observations, that is, several observations taken a short time before, and a like number after, the meridional passage or transit, at times nearly equidistant, observe first with the face of the instrument to the right, and then to the left, by reversion in azimuth, noting the precise time of each observation. Now if, from computation, we have the exact time of the object's transit, by a chronometer shewing either true time, or with a known error and rate, the object's distance from the meridian in time, at the instant of each observation, may be found. This, with the approximate latitude of the place, and the declination of the object, afford, by the formula (6), in page 18, and the aid of Table XVII., data for computing a quantity called the reduction to the meridian, which, *subtracted* from the mean of the observed zenith-distances, will give the apparent meridional zenith-distance of the object. This reduction must be applied with a contrary sign to the altitude. The nearer the observations are taken to the meridian, the less will the accuracy of the results depend upon a true knowledge of the time. To obviate such an error as much as possible, an equal number of observations should be taken nearly equidistant from the meridian, and not extending to more than ten or twelve minutes on each side of it, when the zenith-distance is not less than twenty or thirty degrees, even when taking in quantities of the *second order*. Should the zenith-distance be less than this, in mean latitudes, the time must be limited to five or six minutes; and, when very near the zenith, this method of repetition is not to be recommended.

This instrument may also be very successfully employed to determine the time and the direction of the meridian, either by absolute altitudes and azimuths, or equal altitudes and azimuths, when corrected by the necessary equations, by Table XVIII., for those purposes. The direction of the meridian may be very accurately determined with this instrument, by means of any circumpolar star, especially by the pole-star, when referred to a mark in or near the horizon, as shewn in pages 31, 32, &c.

To insure permanence of position during a course of observations, this instrument is frequently furnished with an under telescope, capable of some degrees of motion, both in a horizontal and vertical direction, till the cross-wires in its focus accurately bisect some well-defined distant object, on which it is firmly clamped at the commencement of a series of observations; and the accuracy of the bisection being examined after their termination, and found perfect, proves the steadiness of the instrument, and no relative motion has taken place during the course of the operations.

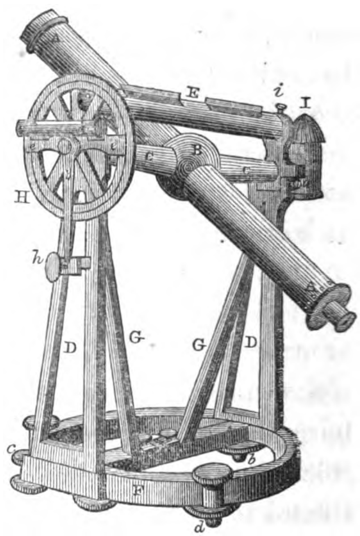
Robinson of London, and Adie of Edinburgh, construct a class of theodolites similar in principle to this instrument, but of smaller dimensions, the divided circles being from five to ten inches in diameter, with three verniers, reading to 20″, 15″, or 10″, according to the size. The arms of the tripod are bent at right angles downwards, so as to raise the horizontal circle sufficiently to admit the conical axis B to descend below instead of above it,—a position perhaps somewhat more convenient. These instruments are, therefore, well fitted to perform all the operations of a theodolite and an astronomical circle with great precision, considering their moderate dimensions and reasonable price. The common method of placing the centre of the vertical axis accurately over a station, is by means of a plumb-line suspended from the under side of the horizontal circle, though, in some of the larger class of instruments, the axis B is hollow in the middle, with two cross-wires adapted to it, cutting each other at right angles in its centre, which, by means of a diagonal eye-piece in its top, is, by a slight motion of the instrument, brought to bisect the centre of the station.

TRANSIT INSTRUMENT.

Description.

A transit-instrument is a telescope, properly placed in the meridian, for the purpose of observing the times at which the celestial bodies pass this circle. The telescope is fitted to an axis, of which the ends, formed into pivots, turn in notches, from their shape called Ys. This axis is made hollow, opposite one of the ends of which is placed a lamp for illuminating the wires in night observations. These wires, generally five in number, are placed in the telescope equidistant from each other, and perpendicular to the horizon, having also a horizontal wire bisecting them at right angles, near or upon which the transits are observed. When properly adjusted, the middle vertical wire coincides with the meridian, and the instant that the centre of any celestial body passes this wire is called its transit. The other parallel wires are intended to correct or verify the observation, by taking a mean between the transits over the first and last, the second and fourth, and comparing it with the third or meridian wire; or, what is more correct, to take a mean of the whole, called the reduction of the wires.

The figure represents this instrument when the telescope varies from eighteen inches to two feet in focal length. The telescope A A consists of two parts, connected together by a sphere B, which also receives the larger ends of the cones C C, placed at right angles to the tube of the telescope, and forming the horizontal axis. This axis terminates in two cylindrical pivots, which rest in Ys fixed at the upper end of the vertical standards D D. One of the Ys possesses a small motion in azimuth, communicated by turning the screw *a*. But that the telescope may move in a vertical circle, the pivots must be precisely in the same level, otherwise the telescope, instead of perpendicularly, will revolve in a plane oblique to the horizon. The levelling of the axis is, therefore, one of the most important adjustments of the instrument, and is effected by means of a spirit-level E, which, for this purpose, is made to ride across the telescope, and rest on the two pivots, and must be removed as soon as the adjustment is made. The standards D D are fixed by screws upon a cast-metal or brass circle F, which rests upon three screws *b*, *c*, *d*, forming the feet of the instrument, and by the motion of which the operation of levelling is performed.



The oblique braces G G are added for the purpose of securing the supports, so that the telescope may have both a free and steady motion. On the extremity of one of the pivots, which extends beyond its Y, is fixed a circle H, which turns with the axis, while the double verniers *e e* remain stationary in a horizontal position, whereof one shews the altitude and the other the zenith-distance at which the telescope is placed. The verniers are both set horizontal by the spirit-level *f*, which is attached to them, and they are fixed in their proper position by a brass arm *g*, clamped to the supports by a screw at *h*. The whole apparatus is moveable along with the telescope, and when the axis is reversed, it can be attached in the same manner to the opposite standard.

Near the eye-end of the telescope is placed a diaphragm in its principal focus, which, in this instrument, has five vertical wires and one or two horizontal wires close to each other, between which the observations are made. The central vertical wire ought to be fixed in the optical axis of the telescope, and perpendicular to the horizontal axis. These wires are visible in the day-time by the light passing down the telescope to the eye; but at night, except a luminous object like the moon be observed, they cannot be seen. In this case they must be illuminated through a hole in one of the pivots of the axis, which admits the light of a lamp placed opposite to it, on the top of one of the standards as shewn at I. This light is directed to the wires by a reflector placed diagonally in the sphere B; which reflector, having a large hole in its centre, admits the rays passing from the object down the telescope to the eye of the observer, who thus sees distinctly both the wires and the object at the same time. The lamp is so constructed that the light may be regulated according to the faintness of the objects, so as not to obscure its feeble rays. The telescope is also furnished with a diagonal eye-piece, by which stars near the zenith may be conveniently observed. The altitude and azimuth circle will, when well constructed and in perfect adjustment, perform the operations of a transit-instrument successfully—a circumstance very important to scientific travellers, who often have not the means of carrying a complete collection of instruments along with them.

Adjustments.

1. *The wires should be set perfectly vertical.*

This is verified by observing that any distant vertical object cut by a wire, does not change its position relative to that wire on moving the instrument up and down. If it does, the wires must be turned till the object is kept upon them when moved through their whole extent, and the adjustment is then complete.

2. *The telescope should have no parallax.*

When any distant object is bisected by the horizontal wire, if, on moving the eye up and down a little, the object should appear to separate from the wire, the instrument is said to have a parallax. This must be corrected by placing the object and eye-glasses at such a distance from each other that their foci may meet at the intersection of the wires. When, as is usually the case, the object-glass has been properly fixed by the maker, the observer has only to adjust the eye-glass.

3. *The line of collimation should be correct.*

This is known by bisecting any object by the meridian wire, and if, on reversing the axis, the object still remains bisected as before, the line of collimation is correct. If not, it must be adjusted by the small screws in the sides of the telescope, carrying the diaphragm near the eye-glass. This is effected by easing one screw and then tightening the other, till the error appears one-half diminished; after which the axis is again reversed, and the operation repeated till the adjustment is properly effected.

4. *To level the axis.*

This is performed by a screw under one of the Ys, which raises or depresses that end of the axis at pleasure, while the true horizontal position is ascertained by the spirit-level.

5. *To bring the telescope into the meridian.*

This is accomplished by a horizontal screw acting on one end of the axis, by which it is moved forward or backward till its proper position is obtained.

6. *To prepare the telescope for observation.*

Slide the eye-piece in or out till the wires are seen distinctly. Direct the telescope to some well-defined object, and turn the milled-head on the side of the transit till the object is seen with perfect distinctness. Place the level on the axis, and bring the bubble to the middle by the screw which elevates or depresses one of the Ys, the axis of the transit will then be parallel to the horizon.

Having brought the object to the central vertical wire by means of the screws, which act horizontally on one of the Ys, observe whether the same point of the object is covered by the wire while the telescope is elevated or depressed, and if not, correct half the apparent deviation by turning round the cell which contains the wires. Now with the wire covering some well-defined distant object, take the instrument out of its Ys and carefully invert it, when, if the wire no longer bisects the same part of the object, correct half the error by means of the screws which act horizontally upon the wires, and the remaining half by the screws which act horizontally upon the Ys. Repeat this operation till the vertical wire covers the same part of the object in both positions of the telescope, and the line of sight will then be perpendicular to the axis.

7. *To elevate the telescope to a given object.*

This operation is performed by computing the altitude or zenith-distance, previously to any observation, and either by the circle on the extremity of the axis in small instruments, or those near the eye-end of the telescope in large ones, elevate it to the proper altitude or zenith-distance, as may be required.

8. *To compute the altitude.*

To the complement of the latitude *add* the declination, if they are of the *same* name, the sum will be the altitude; but subtract it, if of different names, and the remainder will be the altitude; when the object is between the zenith and the pole, of a contrary name to the latitude. If the object is between the zenith and the pole, of the same name with the latitude, the meridian altitude is equal to the sum of the latitude, and the polar distance of the object, when above the pole, but to their difference when below it.

9. *To take a transit.*

With the latitude of the place and the declination of the object, compute its meridian altitude. When it is known by computation, or otherwise, to approach the meridian, elevate the telescope to the given altitude by one or other of the circles for that purpose. Now because the telescope inverts, the object will appear to come into the field of view from the west, and move towards the east. Mark, by the clock or chronometer, the time of transit over each wire, using a dark glass to save the eye when the sun is observed, and tabulate the result in the following manner:—

Edinburgh, 1836.						
Date.	Object observed.	Wires.				
		I.	II.	III.	IV.	V.
		s.	s.	h. m. s.	s.	s.
Jan. 15.	Sun 1 Limb	38.6	52.8	19 44 7.0	21.4	35.8
	Sun 2 Limb	58.6	12.8	46 27.3	41.7	55.9
	α Andromedæ	11.2	26.2	23 59 41.3	56.4	11.6

	h. m. s.
Reduction of sun's 1st limb to III. wire . .	19 44 7.12
Correction of instrument	+ 0.93
Correction of clock	+ 12.07
Apparent right ascension observed . . .	19 44 20.12
In like manner the second limb	19 46 40.26
Mean or that of the centre	19 45 30.19
Reduction of α Andromeda to III. wire . .	23 59 41.34
Correction of instrument	+ 0.47
Correction of clock	+ 12.03
Apparent right ascension observed . . .	23 59 55.84

To bring a transit-instrument into the meridian.

1. Let the time be accurately determined by absolute altitudes near the prime vertical, or by equal altitudes, as shewn in the explanation of Table XVIII. Having got the error of the clock or chronometer to be used in the observation, compute the time of transit of the object to be observed either in mean solar or sidereal time, according to which the time-piece is regulated, making due allowance for error and rate, as shewn in § 15, pages 20, 21, 22, 23, &c., then bring the telescope to the celestial object, when nearly upon the meridian; and by turning the horizontal screw, make the middle wire bisect the object at the instant of its computed transit, and the instrument will be

in the meridian. Should the object be the sun or moon, either limb must be observed; and, allowing for the time which the semidiameter takes to pass the meridian, that of the centre becomes known, or the limb, conversely.

To find the time that any star takes to pass from one wire to another in a transit-instrument, when that on the equinoctial is known.

Rule.—To the log secant of the star's declination, add the logarithm of the time in seconds at the equinoctial, the sum will be the logarithm of the time by the star.

Ex. On the 10th of May 1836, the declination of Capella was 45° 49′ 32″ N., what would be the time of passage of the star from one wire to another, when the time upon the equinoctial was $19^s.64$?

Declination .	45° 49′ 32″ N.	secant	0.156863
Equinoctial time	$19^s.64$	log	1.293141
Star's time .	$28^s.18$	log	1.450004

This may be readily performed by a Table of Natural Secants, like that among my General Tables (XXV.); thus, $19^s.64 \times 1.435 = 28^s.19$. Hence the star's expected time of approach to the other wires becomes known after the first contact is observed.

2. To place a transit-instrument in the meridian by Polaris.

On the 1st of May 1840, let a transit-instrument be placed in the meridian at Edinburgh, in latitude 55° 57′ 24″ N., longitude, in time, $12^m\ 43^s.5$ W.

By the Nautical Almanac, the right ascension of Polaris is $1^h\ 1^m\ 16^s.40$, and declination 88° 27′ 23″.4 N.

Whence, by § 8, page 132.

Latitude . . .	55° 57′ 24″ N.	.	55° 57′ 24″ N.
Star's polar distance	1 32 36 .	.	1 32 36
Sum . . .	57 30 0	Difference	54 24 48

Hence 57° 30′ 0″ is the star's altitude above the pole, or at its upper transit, and 54° 24′ 48″ at its lower transit under the pole. The complements of these will give the zenith-distances.

Now, let the clock be regulated truly to sidereal time, and when it shews $1^h\ 1^m\ 16^s.4$, make the middle wire bisect Polaris, then will the instrument be in the meridian. If the time-piece be regulated to mean solar time, the mean time of transit must be computed as shewn in the explanation of Tables XXVI. and XXVII., illustrated by the example in page 24.

3. To place a transit-instrument in the meridian by a pair of circumpolar stars, differing nearly *twelve hours* in right ascension.

Let t = the time of the first star's upper transit, and t' = that of its lower; also let τ and τ' be the times of the contrary passages of the second star. Now, if δ = the polar distance of the former star, δ' that of the latter, while α is the error in azimuth, and l the latitude.

$$\alpha = \tfrac{1}{2}\{(t-\tau)-(t'-\tau')\} \sec l \sin(\delta \backsim \delta') \quad . \quad . \quad . \quad (1)$$

Ex. On the 1st of January 1828, when the right ascension of the pole-star, by the Nautical Almanac, was $0^h\ 59^m\ 28^s.8$, the polar distance 1° 36′ 9″; the right ascension of ζ Ursæ Majoris was $13^h\ 16^m\ 58^s.3$, the polar distance 34° 10′ 44″; when the clock of an observatory in latitude 52° 25′ 50″ N. was regulated properly, and its error and rate allowed for, the times of four passages taken by the transit-instrument placed a little out of the meridian, but otherwise well adjusted, were as follow:

	h.	m.	s.		h.	m.	s.
Pole-star above .	1	0	0.55 $=t$,	below	12	58	55.47 $=t'$
ζ Ursæ Maj. below	1	16	55.46 $=r$,	above	13	16	58.16 $=r'$
$t-r$ . .	−	16	54.91	$t'-r'=-$		18	2.69
$t'-r'$ . .	−	18	2.69				
$(t-r)-(t'-r')=$	+	1	7.78 $=67^s.78=\Delta$.				

Now, as $t'-r'$, the second interval, exceeds $t-r$, the first, the deviation is towards the *east*, while the difference is $67^s.78$. But, in using formula (1), the error α may be computed either in time or arc, as best suits the observer, or the knowledge he has of the value of a turn of his screws, which he should previously ascertain, at least in an approximate manner. Then, if he wish the correction in time, the constant logarithm will be the arithmetical complement of the log of 2, or 9.698970; if in arcs, the log of $\frac{15}{2}=7.5$, or 0.875061 is the constant.

In Time.			In Arc.	
1. Const. log . .		9.698970	2. Const. log	0.875061
$\delta=$ 1° 36′ 9″	sin	8.446619	. . .	8.446619
$\delta'=$34 10 44	sin	9.749565	. . .	9.749565
$\delta'-\delta=$32 34 35	cosec	0.268876	. . .	0.268876
$l=$52 25 50	sec	0.214868	. . .	0.214868
$\Delta=$ $67^s.78$	log	1.831102	. . .	1.831102
$\alpha=$ $1^s.622$	log	0.210000	$\alpha=24''.33$ log	1.386091

the respective deviations in time and arc towards the east.

4. To place a transit-instrument in the meridian by a pair of high and low stars.

If the difference of the right ascensions of two stars, of which the declinations are δ and δ', be α, and if a transit-instrument be placed s seconds of time out of the meridian, the interval between their transits will be $\alpha+d\alpha$ seconds of time, and $d\alpha$ may be found from the following formula, in which l is the latitude.

$$d\alpha=s\cos l\sin(\delta\backsim\delta')\sec\delta\sec\delta' \quad . \quad . \quad . \quad (2)$$

Let $p = \mathrm{D}\sin(l-\delta)\sec\delta$ . . . . . . (3)

in which δ is negative when of a contrary name to l, and D the deviation or error in the position of the transit.

Now making $n = \sin(l-\delta)\sec\delta$ . . . (4)

then $p = n\,\mathrm{D}$ and $p' = n'\,\mathrm{D}$, and hence

$$\mathrm{D} = \frac{p-p'}{n-n'} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Again, let α be the calculated transit or true right ascension of the first star, and t the observed transit, consequently $p = t - t'$, and α' being the right ascension of the second star $p' = \alpha - \alpha'$, therefore

$$\mathrm{D} = \frac{(t-t)-(\alpha-\alpha')}{n-n'} \quad . \quad . \quad . \quad (6)$$

If the clock does not keep true time, the interval must be corrected for the rate. Then, if D be *positive*, the instrument deviates to the *west;* if negative, to the east, and the correction may be made by the divided head of the adjusting-screw, while the operation is performed as follows:

Ex. 1. On the 1st of April 1840, the following observations were made in latitude 51° 30′ N., and nearly on the meridian of Greenwich.

		h. m. s.		h. m. s.
Observed transit of Regel	$t=$	5 7 34.70	$\alpha=$	5 6 51.30
... ... Capella	$t'=$	5 5 38.08	$\alpha'=$	5 4 53.08
	$t-t'=$	1 56.62	$\alpha-\alpha'=$	1 58.22
	$\alpha-\alpha'=$	1 58.22		
	$\Delta=(t-t')-(\alpha-\alpha')=$	− 1.60		

Whence, to get D, it is only necessary to compute n and n', and, by combining formulæ (4) and (6), the value of D and D′ at the two stars will be found.

$l=51°\ 30'$ N.	.	. .	$l=51°\ 30'$ N.		
$\delta=45\ \ 50$ N.	sec	0.1569	$\delta'=\ 8\ \ 24$ S.	sec	0.0047
$l-\delta=\ 5\ \ 40$	sin	8.9945	$l+\delta'=59\ \ 54$	sin	9.9371
$n=$ 0.142	log	9.1514	n' . .	log	9.9418
$n'=$ 0.874					
$n'-n=$ 0.732	a . c . l	0.1355	. . .		0.1355
$\Delta=-1^s.60$	log	$=0.2041$	. . .		0.2041
$D=-0\ .31$	log	$=9.4910$	$D'=-1^s.91$	log	0.2814

Here D is *negative*, and the deviation of the telescope is towards the *east*, but when positive it is to the west.*

See my Mathematical Tables and General Astronomical Tables on this subject.

	s.			s.
Here $t-\alpha=$.	43.40,	and $t'-\alpha'=$	.	45.00
D′ with Cont. sign	+1.91	D with Cont. sign		+0.31
Error of clock= .	45.21	fast, or	. .	45.31

Or the result may be stated thus:

	h.	m.	s.		h.	m.	s.
Capella's R. A. .	5	6	51.30	Rigel's R. A.	5	4	53.08
D′	—		1.91	D . . .	—		0.31
Time by transit .	5	6	49.39	. . . .	5	4	52.77
... by clock .	5	7	34.70	. . . .	5	5	38.08
Error of clock fast .			45.31	. . . . .			45.31
Correction of clock .			−45.31	in sidereal time.			

If the clock is regulated according to mean time, the interval $t-t'$ must be, by Table XXVI., converted into sidereal.

* If $D=\frac{(\alpha-\alpha')-(t-t')}{n-n'}$, then the sign of D would give the correction of the observed time with its proper sign, which would be the contrary of those stated above.

Ex. 2. On the 24th of April 1828, the following observations were made at Paris on γ Ursæ Majoris above the pole, and on β Cephei under the pole.

		h. m. s.		h. m. s.		
γ Ursæ Maj.	$t=$	11 44 13.80,	$\alpha=$	11 44 47.10,	$n'=$	-0.176
β Cephei	$t'=$	9 25 54.80,	$\alpha'=$	9 26 24.49,	$n=$	$+2.542$
	$t-t'=$	2 18 19.00,	$\alpha-\alpha'=$	2 18 22.61,	$n'-n=$	-2.718
	$\alpha-\alpha'=$	2 18 22.61				
	$\Delta=$	$-$ 3.61 to the right.				

Hence making $x=\frac{-3.61}{-2.718}=+1^s.33$, we have

$nx=2.542\times 1^s.33=+3^s.38$, and $n'x=-0.176\times 1^s.33=-0^s.23$

	h. m. s.		h. m. s.
Whence to $\alpha-12^h=$	9 26 24.49,	and $\alpha'=$	11 44 47.10
Apply nx $=+$	3.38,	and $n'x=-$	0.23
Transit .	9 26 27.87	. .	11 44 46.87
t' .	9 25 54.80	t . .	11 44 13.80
Clock slow of S. T.	33.07	. .	33.07

Remark 1. If, when a circumpolar star is observed between the pole and the zenith of the upper meridian, the same formulas apply, since n is then negative, because d exceeds l.

2. If the transit is taken between the pole and the horizon, the same formulas will still answer, by diminishing the right ascension of the star by 12^h, and changing the sign of d. The deviation of the telescope pointing to the north is still reckoned to the right, when x is positive; but here this side is found towards the east. The contrary takes place when x is negative. When two circumpolar stars are observed, the same remark is applicable to both.

3. When the same star is observed at both passages, superior and inferior, the preceding rule is applicable to both,

the right ascension of the star must be diminished by 12^h for the inferior passage, and the sign of *d* must be changed.

Since the sheet containing page 58 was thrown off, it has been discovered that formula (4) should have been

$$\text{Sin}\,\tfrac{1}{2}\,H = \left\{ \tfrac{1}{2}\left(1 \pm \sec D_{,}\left[\sin(D + D_{,})\sin(D - D_{,})\right]^{\frac{1}{2}}\right)\right\}^{\frac{1}{2}}.$$

By reflecting on the steps of the investigation, it appears that

$$5.\ \text{Sin}\,(H - 90^\circ) = \sec D_{,}\left\{\sin(D + D_{,})\sin(D - D_{,})\right\}^{\frac{1}{2}}.$$

Because $D_{,}$ is always a small arc, its secant differs little from radius, therefore its effect will be nearly insensible; hence,

1. $H - 90^\circ = 2^\circ\ 7'\ 42''.3$	sine	8.569835
	log sec $D_{,}$ . .	0.000051
2. $H - 90^\circ = 2^\circ\ 7'\ 43''.2$		8.569886

The second value, therefore, exceeds the first by $1''$ only, a difference of little consequence in this problem.

EXPLANATION OF THE TABLES.

TABLE I. *Depression or Dip of the Horizon.*

The dip of the horizon is the angle contained between a line perpendicular to the plumb-line, passing through the eye of the observer, elevated above the level of the sea, and a line from his eye to the visible horizon when they are in the same vertical plane. This table contains the apparent dip answering to a free unobstructed horizon, diminished by 0.08 of itself, or of the intercepted arc for the effects of refraction.

1. The numbers in the table corresponding to the height of the eye of the observer, is to be *subtracted* from the observed altitude when taken by the fore observation with Hadley's quadrant and similar instruments, but added to it in case the altitude be taken by the back observation.

2. This has been the principal use to which analogous tables have hitherto been applied, but it may be often advantageously employed for other purposes, which has been an inducement to extend it a little beyond the usual limits. Since the *true* dip has been diminished by 0.08 or about $\frac{1}{12}$ of itself to reduce it to the *apparent*, it consequently follows that if the apparent dip be increased by double of 0.08, or 0.16, equal to $\frac{1}{6}$ of itself nearly, the result will be the distance of the visible horizon in geographical miles.

3. If the *apparent dip* be measured with a good theodolite or astronomical circle, the corresponding height of the in-

strument above the sea, will be found by the table with as much accuracy as the nature of the horizontal refraction will admit.

EXAMPLES.—1. To the height of the eye, 16 feet in the first column, will be found 3′ 56″, the dip in the second.

2. To the height of the eye, 500 feet,

there will be found . . .	22′
To this add $\frac{1}{6}$ of itself . . .	$3\frac{2}{3}$
Sum or distance of the horizon .	$=25\frac{2}{3}$ miles.

3. From a point on Inchcolm the author observed the depression of the horizon of the sea down the Firth of Forth to be 8′ 21.″2; required the height of the instrument above the sea?

By the table to dip . .	=8′ 14″	the height is	70 feet.
Proportional part to . .	7.2	.	+ 2.1 feet.
Height of instrument for .	8 21.2	=	72.1 feet.

TABLE II. *Correction of the Apparent Altitudes of the Sun and Stars.*

In this table the altitude is found in the first column, the star's correction or *mean refraction* in the second, and the difference between the mean refraction and the sun's mean parallax, constituting the sun's correction in the third.

EXAMPLES.—1. Required the correction of the altitude of a star which was observed to be 22° 30′?

ANSWER . . . . . . 2′ 20″.0.

2. Required the sun's correction at an altitude of 31° 20′?

ANSWER . . . . . . 1′ 36″.3.

These are the true corrections when the English barometer stands at 30 inches, and Fahrenheit's thermometer at 50°, and are always to be *subtracted* from the apparent altitude or added to the apparent zenith distance to obtain the true.

TABLE III. *To correct the Mean Refraction.*

When the barometer differs from 30 inches, and the thermometer from 50°, the mean corrections, as above, may be reduced for the effects of pressure and temperature by this table with sufficient accuracy, when altitudes are taken with the ordinary theodolite or sextant. These corrections must be applied according to the signs in the table. Thus, in the first example to the preceding table, let the observed height of the barometer $b=29^{in}.57$, and the temperature by Fahrenheit's thermometer $t=84°$, then

To the mean refraction formerly found . . .	2′	20″
There must be applied for the altitude 22° 30′, and $b=29^{in}.57$	−	2
For altitude 22° 30′, and $t=84°$	−	10
True refraction	2	8

or the star's correction to be subtracted.

In like manner for the second example,

To the mean correction	1′	36″.3
For altitude 31° 20′, and $b=30^{in}.28$	+	1 .0
For altitude 31 20 and $t=30°$ Fahrenheit .	+	4 .0
The sun's true correction	1	41 .3

and so on in similar cases.

TABLE IV. *Correction of the Apparent Altitude of the Moon.*

This table contains the difference between the moon's parallax in altitude and the mean refraction, and must be always *added* to the apparent altitude to obtain the time. To the moon's apparent altitude in the first column on the left, and under the minutes in the moon's horizontal parallax at the top, will be found the correction for the nearest less degree of altitude and minute of parallax; and the proportional parts for minutes of altitude and seconds of parallax, are found in the two adjacent right hand columns, taking care not to neglect the parts to *o′* of altitude, as for the sake of the convenience of having all the parts *additive*, the construction of the table requires.

Example 1.—Let the moon's apparent altitude be 32° 40′, and the equatorial horizontal parallax 58′ 32″; required the true correction when the barometer stood at 29.6 inches, and Fahrenheit's thermometer at 72°?

To app. altitude 32° and horizontal parallax 58′ correc. .	+47′ 10″
To app. altitude 0° 40′ proportional parts . .	+ 10
To seconds of parallax 32″	+ 27
True correction for $b=30°$, and $t=50°$ . .	47 47
To $b=29^{in}.6$ and altitude 32° 40′ correction . .	+ 1
To $t=72°$ and altitude 32° 40′ correction . .	+ 4
True correction	47 52

for real temperature and pressure, where, according to the remark at the foot of Table III, the corrections depending upon *b* and *t* have been applied with signs *contrary* to those marked in the table.

Table V. *Mean Refractions.*

This table contains the *logarithms* of the mean refractions at 30 inches of the English barometer, and 50° of Fahrenheit's thermometer. It is succeeded by Tables VI., VII., VIII., IX., and X., to reduce it to any other pressure and temperature, either for the English barometer and Fahrenheit's thermometer by the first three auxiliary tables, or for the metrical barometer and centigrade thermometer by the two last, in which the logarithms for *τ* and *t* are, as is frequently the case, united in one with the argument *t*, a method that in general cannot sensibly affect the accuracy of the results. Logarithmic tables of refraction are used by all astronomers where extreme precision, combined with facility of calculation, are required.

EXAMPLES—1. Let the zenith-distance θ be 68° 55′ 36″, the barometer $b=28.80$ inches, and the thermometers τ and t each 65° Fahrenheit; required the refraction?

For θ =	68° 40′	log $\delta\theta$ . . .	2.17171
Prop. part for	15.6 = 37.1 × 15.6 =	. .	+ 579
b =	28^{in}.80	log (Table VI.) . .	9.98227
τ =	65°	log (Table VII.) . .	9.99935
t =	65°	log (Table VIII.) . .	9.98663
r =	2′ 19″.87 = 139″.87	. . .	log 2.14575

2. Let $\theta=87°\ 42'\ 10''$, $b=29^{in}.50$, τ and $t=35°$; required r, the refraction?

For θ =	87° 40′ 0″	log $\delta\theta$ . . .		3.00522
Prop. parts for	2 10	. . .		392
b =	29^{in}.50	log as before . .		9.99270
τ =	35°	log . . .		0.00065
t =	35°	log . . .		0.01379
r'' =	. . .	.	1038″.20	log 3.01628
$\frac{d\delta\theta}{d\tau}\times(35°-50°)=-0''.591\times-15°$ =		.	+ 8 .86	
$\frac{\delta d\theta}{dp}\times(29^{in}.5-30^{in})=+1''.04\times-0.5$ =		.	− 0 .52	
r =	. . . .	.	1046 .54	= 17′ 26″.54
r as observed by Brinkley	. . .			17 26 .50
Difference between theory and observation		.		+ 0 .04

3. Let $\theta=88°\ 24'\ 9''.7$, the metrical barometer $b=755$ millimetres, and τ and t each 8°.75 centigrade; requird the refraction?

θ =	88° 24′ 9″.7	log $\delta\theta$		3.08937
b =	755$^{m.m}$	log (Table IX.)		9.99599
t =	8°.75 cent.	log (Table X.)		0.00216
r''	. . . .	.	1223″.23	log 3.08752

$$1223''.23 \log 3.08752$$

$$\frac{d\,\delta\,\theta}{d\,\tau} \times (8^\circ.75 - 10^\circ) = 0''\!\cdot\!91 \times -1.25 \times 1.8 \quad = +2.05$$

$$\frac{d\,\delta\,\theta}{d\,p}(755^{\mathrm{m.m}} - 762^{\mathrm{m.m}}) = +1.62 \times -7 \div 40^{\mathrm{m}} \quad = -0.28$$

r	$=1225.00=$	$20'\ 25''.00$
r from observation by Plana		20 24 .30
Difference of theory from observation . .		+ 0 .70

From these instances it is evident that the table gives the value of the refraction with great accuracy. It must be added to the zenith distance and subtracted from the altitude. The reason that the first small correction of r is multiplied by $1^\circ.8$ is, that 1° centigrade is equal to $1^\circ.8$ Fahrenheit, that to which the corrections in the table are adapted, and as $39^{\mathrm{in}}.371$ are equal to a metre, the second must be divided by this or even by 40, as sufficiently accurate.

Table XI. *Logarithms to compute the value of the Coefficient of Terrestrial Refraction.*

In the practice of Trigonometrical Levelling it is of the utmost importance to get the value of terrestrial refraction truly. Hitherto it has generally been the practice to determine occasionally from numerous observations, its mean value by reciprocal and simultaneous measures, and to employ it either exactly as obtained, or under assumed variations in all cases. Reflecting on the inaccuracy of this plan, I endeavoured to investigate a formula which would give, with as much precision as the nature of the case seemed to admit, the value of the coefficient of terrestrial refraction, and from which the present table has been derived.*

Example.—Let the barometer $b=29.75$ inches, the attached

* See the Edinburgh New Philosophical Journal for April 1841.

thermometer $\tau=64°$, and the detached $t=64°$ Fahrenheit; required n the coefficient of refraction?

For b	=	$29^{in}.75$ and $t=64°$ (Table XI.) .	log 7.44997
τ	=	64° log (Table VII.) . .	9.99940
t	=	64° log × 2 (Table VIII.) . .	9.97502
b	=	29.75 log	1.47349
n	=	0.07905 log . . .	8.89788

TABLE XII. *Parallax of the Sun in Altitude or Zenith Distance.*

The parallax of the sun on the first day of each month at the top, and to every third degree of altitude in the left hand column, or zenith distance on the right, will be obtained from this table by inspection, and for any intermediate day or degree it may be readily found by interpolation, as will be subsequently exemplified.

TABLE XIII. *Parallax of the Planets in Altitude or Zenith Distance.*

This is precisely similar to the last, and is used in the same manner.

EXAMPLE. Required the parallax of Venus at an altitude of 30° on the 1st of December 1840, when the horizontal parallax was 12″.6?

		″	″
To altitude 30° and parallax	.	10.	8.7
... ... 30	. . . .	2.	1.7
... ... 30	. . .	0.6	0.5
To altitude 30 and horizontal parallax		12.6	10.9

Hence the parallax in altitude is found to be 10″.9.

TABLE XIV. *Augmentation of the Moon's Semidiameter in Altitude or Zenith-Distance.*

With the moon's semidiameter at the top, and altitude on the left, or zenith-distance on the right-hand column, will be found the augmentation to be added to the semidiameter on that account.

TABLE XV. *Reduction of the Moon's Parallax on the Spheroid.*

With the moon's equatorial horizontal parallax at the top, and the latitude on the left-hand column, the reduction to be *subtracted* from the moon's equatorial horizontal parallax, to reduce it to the given latitude, will be found.

TABLE XVI. *Reduction of the Latitude on the Spheroid.*

With the observed latitude on the left, the reduction will be found on the right, to be subtracted from the observed latitude, to get the reduced latitude, or that referred to the centre of the earth, considered as a spheroid of $\frac{1}{300}$ of compression.

TABLE XVII. *Reduction to the Meridian.*

The method of determining the latitude by repeated observations near the meridian, makes the smaller classes of instruments much more efficient than they otherwise would be. Indeed it renders them much more nearly equal to the larger classes of instruments, such as the mural circles, than could have been anticipated. There are various methods of accomplishing this. In many cases the numbers in the table are given in seconds of arc and decimals, in others they are merely versines. The late Dr Thomas Young first gave, I believe, a small table of versines for this purpose, similar to ours. I have, however, extended the numbers in the column titled V to one place more than his, which includes quantities to the first order only. To this I have added another column *v*, entirely omitted in Young's, embracing quantities to the second order in the formula, which cannot be dispensed with when the zenith-distance is small, not greater than about 10°, and the time extending to about ten minutes from the meridian, &c.

In this way the numbers are all integers, which, by those not very familiar with decimal fractions, render them more easily manageable; while, by means of the logarithms corresponding to the number of observations at the end of the table, the results will be readily converted into seconds of arc. The observations are generally taken in pairs, and therefore logarithms of the even numbers will generally be enough, though the logarithm for one observation, which may occasionally be required, is also given.

EXAMPLE 1. Required the values of V and v for $12^m\ 36^s$ from the meridian?

By the table, under 12^m at the top, and opposite 36^s in the left-hand column, will be found $V = 15109$, and $v = 2283$.

2. Let the time from the meridian exceed the limits of the table, then the value of V to *half* the time being *quadrupled*, will be the value to the whole time nearly; and the value of v to half the time multiplied by 16 will give the value of v also for the whole time nearly.

Thus, let the time be $20^m\ 40^s$, then to one-half of this, or $10^m\ 20^s$, $V' = 10163$ and $v' = 1033$, whence $V = 10163 \times 4 = 40652$, and $v = 1033 \times 16 = 16528$ nearly. The true values of these, by direct calculation, being $V = 40630$ and $v = 16508$. The differences would not materially affect the accuracy of the final result in any ordinary case, especially when combined with a number of other values near the meridian within the limits of the table. It would not be desirable, however, to extend observations beyond the limits of the table; and it would be conducive to accuracy to take always an equal number of observations nearly equidistant, on each side of the meridian, to avoid, as far as possible, the effects of any little uncertainty in the time.

If the sun be the object, the observations should be taken by a watch shewing mean solar time,—if a star, by a watch shewing sidereal time.

If the sun be the object, and the watch regulated to sidereal time, V must be multiplied by 0.9945466, the square of the number to convert sidereal into mean solar time, of which the logarithm is 9.997625; and if the watch be regulated to mean solar time when a star is observed, V must be multiplied by 1.0054833, of which the log is 0.002375, to convert the effect from mean solar into that from sidereal time.

When the watch does not go accurately to either times, the value of V must be further multiplied by $1 + 0.00002315\,r$, whose log is $0.000010053\,r$, in which r is the rate of the watch, reckoned PLUS when LOSING, and *minus* when *gaining*. When r is negative, the arithmetical complement of the log denoted by $0.000010053\,r$ must be taken.

If these rates be small, and the distance from the meridian moderate, their effects will hardly be sensible.

When the zenith-distance exceeds 30°, the *first term* of formula (6), page 18, will be sufficient; and if the object is below the pole, the reduction must be applied with a contrary sign.

On applying this table in formulæ (11) and (12), page 32, $d\,l$ is *positive* when the star is *above* the pole, negative when below it; $d\,m$ is *positive* when the star is in the semicircle *farthest* from the referring lamp or staff, negative when nearest. Consequently, in page 33, line 12 from the bottom, the reduction to the centre was really $-33''.18$, but to render $d\,m$ positive, so that all the columns might be *added*, the double of $-1''.26$, the mean of these, or $-2''.52$, was added to $-33''.18$, making it $-35''.70$, which artifices are admissible, at the option of the computer, when they conduce to facility or convenience.

TABLE XVIII. *Logarithms to compute the Equation to Equal Altitudes and Equal Azimuths.*

The first column in this table contains the elapsed time between the observations, and is the common argument to the other three columns A, B, C. The two first, A and B, are employed to compute the equation to equal altitudes in *seconds of time*, and C to compute the equation to equal azimuths in *seconds of arc.*

The computation of the equation to equal altitudes is performed by the following rules.

1. To the log A, from Table XVIII., add the log tangent of the latitude, the log of the hourly variation of the sun's declination from the Nautical Almanac, to be marked positive, or +, when the polar distance is increasing, and negative, or −, when decreasing; the sum of these three logarithms will be the log of part first of the equation for *noon.* The signs must be reversed for midnight.

2. To log B add the log tangent of the declination to be reckoned *positive*, if the polar distance is *less* than 90°, but negative, if greater; and the log of the sun's hourly variation reckoned positive, if the polar distance is decreasing, but negative, if increasing; the sum will be the log of the second part of the equation for noon or midnight.

3. To the log C, from Table XVIII., add the log cosecant of the latitude, and the log of the sun's hourly motion from the Nautical Almanac, the sum will be the equation to equal azimuths, in seconds of arc, to be allowed to the left of the meridian indicated on the horizontal circle for the noon of the same day, when the polar distance is decreasing, but to the right if increasing. The signs must be reversed for midnight, or the correction for the meridian must be allowed to the right when the polar distance is decreasing, but to the left when increasing.

EXAMPLE 1. At Madeira, in latitude 32° 38′ 25″ N., longitude $1^h\ 7^m\ 35^s$ W., on the 8th of August 1840, at $9^h\ 55^m\ 34^s.2$ forenoon, and $4^h\ 26^m\ 26^s.5$ afternoon, by chronometer, the sun had equal altitudes; required the time of apparent noon?

	h.	m.	s.		h.	m.	s.		h.	m.	s.
½ (16	26	26.5	+	9	55	34.2)	= Approx. noon =	13	11	0.35
E. T. = (16	26	26.5	−	9	55	34.2)	= . .	6	30	52.30

Log A . .	+9.4599	Log B . .	+9.2779
Lat. 32° 38′.4 N. tan	+9.8065	Dec. 16° 4′ N. tan	+9.4594
$\delta d = +43''.17$ log	+1.6352	. . . . .	−1.6352
1st part $+7^s.97$ log	+0.9016	2d part $= -2^s.36$ log	−0.3725
2d part −2 .36			

E. E. A. $+5.61 + 13^h\ 11^m\ 0^s.35 - 12^h$ (App. noon) $1^h\ 11^m\ 5^s.99$

2. At the same place in August 1840.

August 8th, P. M. and August 9th, A. M.

	h.	m.	s.		h.	m.	s.		h.	m.	s.
½ (4	29	56.5	+	21	59	37.5)	= App. Midn. =	13	14	47.00
E. T. (21	59	37.5	−	4	29	56.5)	= . .	17	29	41.00

Equation to equal altitudes for Midnight.

Log A . .	+9.8895	Log B . .	−9.7082
Lat. 32° 38′.4 N. tan	+9.8065	Dec. 15° 56′ N. .	+9.4556
$\delta d = -43''.45$ log	−1.6380	. . . .	−1.6380
1st term $-21^s.58$ log	−1.3340	2d term $+6^s.34$ log	+0.8018
2d term + 6 .34			

E. E. A. $-15.24 + 13^h\ 14^m\ 47^s.00 - 12^h$ (Midn.) $= 1^h\ 11^m\ 31^s.76$

3. On the 28th of February 1840, in latitude 55° 57′ N., longitude $12^m\ 43^s.5$ W., in an interval of $5^h\ 30^m\ 0^s$, the sun had equal altitudes when the azimuth circle read 130° 10′ 15″ and 32° 36′ 15″, and consequently the middle point = ½ (130° 10′ 15″ + 32° 36′ 15″) = 81° 23′ 15″; required the true meridian point?

To interval $5^h\ 30^m\ 0^s$ log C . . .	0.6202
Latitude 55° 57′ N. log secant . .	0.2519
$\delta d = 56''.69$ log	1.7535
E. E. Az. 7′ 2″.3 = 422″.3 log	2.6256

		°	′	″
Hence if from	. . . . .	81	23	15.0
There be subtracted	. . .	—	7	2.3
The remainder	. . . . .	81	16	12.7

is the reading of the instrument when set to the true meridian.

TABLE XIX. *Logarithms to convert Feet on the Surface of the Terrestrial Spheroid into Seconds of Arc, and conversely.*

This table, of great use in Trigonometrical Surveying, contains the logarithms of the reciprocals of the radii of curvature in any given direction multiplied by the arc, equal to the radius in seconds. Log M are those on the meridian, Log P those on an arc perpendicular to the meridian, and Log O those in any direction indicated by the azimuth α or Z. The differences for each degree in M and P are given, to interpolate more easily for minutes of latitude.

EXAMPLES. Required the log M for latitude 51° 13′.5, the log P for 50° 58′.3, and the log O for latitude 56° 4′.5, and α or Z = S. 106° 46′.4 W.

1. To latitude 51° 0′ log M	. .	7.9940850
Prop. parts for 13.5	. . . .	— 167
Log M to latitude 51° 13′.5	. .	7.9940683
2. To latitude 50° 0′ log P	. . .	7.9929588
Prop. part to 58.3	. . . .	— 241
Log P to latitude 50° 58′.3	. . .	7.9929347
3. To lat. 56° and Z = 100° log O	.	7.9928405
Prop. parts to 4′.5 of latitude	. .	— 19
Prop. parts to 6°.8 of azimuth	. .	+ 536
Log O to lat. 56° 4′.5 and Z + 106° 46′.4		7.9928922

The numbers from Tables XX. and XXI. are taken out in the same manner.

TABLE XXII. *Reduction of* λ *to* l.

This table, computed from the formula $p''^2 \frac{1}{2} \sin 1'' \tan \lambda$, in which λ is the latitude of the foot of the perpendicular arc from the given station on the meridian passing through that required, and p'' the length of that arc itself in seconds of arc, gives to λ at the top of the page, and p' in the left-hand column, in minutes, a small correction within its limits to be subtracted from λ to give l, the true latitude of the required point, derived trigonometrically from the first. If the quantity is not got at sight, it may be easily found by interpolation.

TABLE XXIII.

This table is the same in principle as the last, but extended to every degree through the British Islands, for the purpose of facilitating calculations made within its range.

EXAMPLE. Required the reduction of λ to l, when $\lambda = 57° 51' 4.5''$, and $p'' = 24' 36''$?

$\lambda = 57°$ and $p'' = 24'$ give	$-7''.73$
Prop. parts for $51'$ of λ	$-0\ .27$
... for $36''$ of p''	$-0\ .40$
Sum	$-8\ .40$
λ	$57° 51'\ 4\ .50$
l the true latitude	$57\ 50\ 56\ .10$

TABLE XXIV. *To reduce a Base at the level of the Sea to any height above it, or from any height above the Sea to its level.*

EXAMPLE 1. Required the length of the chord K, when the arc a is 164045 feet, and height above the sea $h = 6562$ feet?

Log a	$+5.2149630$
For $h = 6000$ feet, into	$+0.0001246$
500	$+\ 104$
62	$+\ 13$
For $a = 100000$ feet, $p\,a^2$	$-\ 4$
64000 ft. $\Delta_, + 0.64 - \text{Eq.}\ \Delta_2 = -13 + 0.64 - 1 =$	$-\ 7$
Log K at the height h	5.2150982

2. The base, on Hounslow-Heath, was found to be by

Roy	27404.0843 feet.
Mudge	27404.3155
Mean	27404.1999
Log of this	4.4378172
For $h=54$ feet above the sea . .	− 11
For $a=27404.2$ feet, $p\,a^2$ . .	+ 0
Log K at the level of the sea . .	4.4378161

This table, therefore, serves to reduce bases from the level of the sea to great heights, for the purpose of accurate trigonometrical levelling, or for reducing a measured base to the level of the sea, in order to extend a series of triangles at that level over a tract of country.

TABLE XXV. *The measure of one Minute of Arc in feet at each degree of latitude.*

As the latitudes and longitudes of a number of places throughout the British Isles will shortly be made known in the volumes of the Trigonometrical Survey, then, by taking a few angles, and either measuring a base carefully, or, if possible, selecting a distance from the survey, the position of any particular point at a moderate distance may be readily fixed by means of this table.

EXAMPLE. In the Island of Iona, Carn Cul ri Eirn is south of Carn Dunii 9955 feet, and west of it 8111 feet; required the latitude and longitude of Carn Cul ri Eirn, those of Dunii being 56° 20′ 33″ N., longitude 6° 23′ 36″ W.?

By the table, 1′ of latitude at 56° 20′ is 6087.2 feet, therefore $9955 \div 6087.2 = 1'.65 = 1'\ 39''$ S. Hence $56°\ 20'\ 33'' - 1'\ 39'' = 56°\ 18'\ 54''$ N., the latitude of Carn Cul ri Eirn.

In like manner, the length of a minute of longitude is 3381.3 feet; hence $8111 \div 3381.3 = 2'.4 = 2'\ 24''$, therefore

6° 23′ 36″ + 2′ 24″ = 6° 26′ 0″ W., the longitude of Carn Cul ri Eirn.

TABLE XXVI. *To convert Mean Solar into Sidereal Time.*

This table gives the quantity to be ADDED, as expressed at the top of the table, or the acceleration, as it is generally called, at the bottom, to be added to any quantity of mean time, to reduce it to sidereal.

TABLE XXVII. *To convert Sidereal into Mean Solar Time.*

This table gives a portion of time to be subtracted (and therefore called retardation) from a known portion of sidereal time, to reduce it to mean time.

The conversion of the time of any phenomenon recorded in sidereal time, into that by mean solar time, and conversely, may be performed in the following manner:—

Let m be the mean solar time at the place of observation.
s the corresponding sidereal time.
σ the sidereal time at mean noon on the meridian of the place of observation, deduced from the Nautical Almanac, page II. of each month, by Table XXVI., the reduction from which is + in west longitude, and − in east.
a the acceleration for mean time, m, by Table XXVI.
α the acceleration of the fixed stars for the sidereal time, $s-\sigma$, from Table XXVII., then

$$m=(s-\sigma)-\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

$$s=\sigma+m+a \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

EXAMPLES.—1. At what mean solar time did α Aquilæ pass the meridian of Inchkeith on the 21st of August 1840?

	h.	m.	s.
σ, the sidereal time at Greenwich mean noon, . August 21. 1840, by Naut. Almanac, .	9	59	29.28
Reduction for longitude $12^m\ 32^s$ W. (T. XXVI.)	+		2.06
σ at Inchkeith mean noon	9	59	31.34
s the R. A. or Sid. T. of transit of star . .	19	43	2.08
$s-\sigma$, or difference	9	43	30.74
α to this difference (Table XXVII.) . . .	−	1	35.60
$m=(s-\sigma)-\alpha=$ mean solar time . . .	9	41	35.14

2. On the 14th of August 1840, on the meridian of Paris, in longitude $9^m\ 21^s.33$ E., at $22^h\ 22^m\ 13^s.4$ mean solar time, what was the sidereal time?

	h.	m.	s.
σ at Greenwich mean noon, Naut. Almanac .	9	31	53.41
Reduction to $9^m\ 21^s.3$ E. (Table XXVI.) .	—		1.54
σ at Paris mean noon	9	31	51.87
m	22	22	13.40
a to $22^h\ 22^m\ 13^s.4$ (Table XXVI.) . . .	+	3	40.49
$s=\sigma+m+a=$sidereal time	7	57	45.76

Table XXVIII. *To convert Degrees, Minutes, and Seconds of Arc on the Equator into Sidereal Time.*

Example. What is the sidereal time corresponding to 56° 38′ 40″?

	°	′	″		h.	m.	s.
To	55	0	0	sidereal time . .	3	40	0
	1	0	0	. . . . . .		4	0
		38	0	. . . . . .		2	32
			40	. . . . . .			2.667
To	56	38	40	sidereal time . .	3	46	34.667

Table XXIX. *To convert Sidereal Time into Degrees, Minutes, and Seconds of the Equator.*

Example.—Required the arc of the Equator corresponding to $5^h\ 48^m\ 36^s.48$ of sidereal time?

	h.	m.	s.		°	′	″
To	5	0	0	the arc is . .	75	0	0
...		48	0	... . .	12	0	0
...			36	... . .		9	0
...			0.4	... . .			6.0
...			0.08	... . .			1.2
To	5	48	36.48	the arc is . .	87	9	7.2

Table XXX. *Diurnal Variations.*

As in the Nautical Almanac, and other Ephemerides, the places of many of the celestial bodies are given for 24^h or 12^h, this table will serve to reduce them to any intermediate time very readily.

EXAMPLE.—What was the sun's longitude at Edinburgh on the 21st of August 1840, at $9^h\ 41^m\ 35^s$, or at $9^h\ 54^m\ 18^s$ on the meridian of Greenwich?

	°	′	″
Longitude 21st, at mean noon	148	25	19.0
... 22d, ... ,	149	23	11.1
Variation in 24^h	+0	57	52.1

	°	′	″
Now to longitude 21st	148	25	19.0
Prop. parts for $9^h\ 54^m\ 18^s$	+	23	52.5
Longitude required	148	49	11.5

In those cases where there are differences given in the Nautical Almanac for one hour, ten minutes, &c. the reductions by this table is then unnecessary, because, when the time of observation is known, the proportional parts may be obtained by multiplying the variation by the hours and parts of an hour, &c. Thus at Lamlash, in the Island of Arran, in longitude 20^m 30 W., on the 11th of August 1836, at $6^h\ 21^m\ 30^s$ of Lamlash time, or adding the longitude ($20^m\ 30^s$), because it is west, and the sum $6^h\ 42^m = 6^h.7$ is the Greenwich time, at which a series of observations on the sun were made to determine the true time and error of the chronometer. For this time then the sun's declination, equation of time, &c. are required by the Nautical Almanac.

	°	′	″	
August 11th 1836, at Greenwich mean noon, the sun's declination is	15	12	51.8	N.
Reduction $= -45''.02 \times 6^h.7 = -301''.6 = -$.		5	1.6	
True declination for Lamlash . .	15	7	50.2	N.
	90	0	0.0	
North polar distance	74	52	9.8	

When the latitude and declination are of the same name, the declination must be subtracted from 90° to get the polar distance, but must be added to it when they are of contrary names. In the same way the equation of time, &c. may be found.

TABLE XXXI. *Shewing the lengths of horizontal lines equivalent to the several ascending and descending planes, the length of the plane being unity; in reference to the different classes of engines, including the gross weight with engine and tender.*

The first part of this table was drawn up, I believe, by Mr Barlow of Woolwich, for the Railway Commission appointed to examine the different railways submitted to Parliament, and its use has been shewn in the article on Railways immediately preceding.

In the second part are also given similar results from experiments to which I had access, and the velocities in different slopes from experiments lately made by Dr Lardner, the value of which rests on his authority.

TABLE XXXII.

This table gives the content in cubic yards of any cutting for one imperial chain of 100 links, or 66 feet, or 22 yards in length, and varying in depth from 1 to 50 feet on a base or formation-level of 30 feet, with the different slopes 1 to 1, $1\frac{1}{2}$ to 1, and 2 to 1, that is, 1 horizontal to 1 perpendicular, $1\frac{1}{2}$ horizontal to 1 perpendicular, and 2 horizontal to 1 perpendicular, which include most of the slopes generally required. Thus clay, chalk, &c. will stand on the sides of cuttings at 1 to 1, gravel $1\frac{1}{2}$ to 1, sand, &c. 2 to 1, and the cuttings must be made accordingly. To this formation-level of 30 feet, will likewise be found half the width at the top or surface, when the cutting varies from 1 to 50 feet at the different slopes mentioned in the table. There is also added another column giving the effect of a change of 1 perpendicular foot in breadth, in order to adapt the table to different bases either above or below 30 feet. If the base exceed 30, the number of yards in this column, multiplied by the number of feet *greater* than 30, gives a correction to be *added* to the content from the preceding column, but to be subtracted if less. The half-width must also be corrected by increasing or diminishing the change made on the base, in the ratio of the slopes.

If the length of the cutting differ from one chain, the number from the table must be multiplied by the number of chains considered an integer, and the links a decimal, the product will be the content in cubic yards. This table is computed on the supposition that the *depth* is uniform, or nearly so, in each portion for which the calculation is made. If *it* varies rapidly, the portions to which it is applied must be diminished to a few links. In this manner the table will suit most ordinary cases likely to occur. If not, then Mr Macniell's tables must be applied, which are well adapted to all sorts of cuttings, but are unfortunately rather expensive for common use.

Though the slopes in the table are those most commonly used, yet they may sometimes fall between or beyond them. Then to the width at the base in feet, add the horizontal length of the side of the triangle formed by the slope, multiply the sum by the depth of the cutting, and also by the length, all in feet, the product divided by 27, will give the content in cubic yards.

It is to be remarked that the depth, multiplied by the slope, gives the side of the triangle to be added to the base, to give the mean breadth, which, multiplied by the depth, gives the area of the section, and this by the length to give the content of the cutting.

Examples—1. Let the length of a cutting be 3.75 chains, the depth 40 feet, the base or formation-level 30 feet, with slopes $1\frac{1}{4}$ to 1, there will be found in the table 8800 cubic feet for 1 chain.

Therefore $8800 \times 3.75 = 33000$ cubic yards, the quantity of cutting required.

2. For a height or depth of 40, and a base likewise of 40 feet, multiply the number under content for 1 perpendicular foot in breadth by 10, the product will be the number of cubic yards to be added to the number for 30 in the table, to give that for 40 feet of base, thus :—

$$8800.00 + 10 \times 97.77 = 8800.00 + 977.7 = 9777.7$$

cubic yards for 1 chain.

This last, multiplied by the length 3.75 chains, will give

$$9777.7 \times 3.75 = 36666.37 \text{ cubic yards.}$$

3. To compute the content for 1 chain in length for slopes not given in the table, suppose we have a cutting with a width of base or formation-level of 28 feet and a depth of 16 feet, the sides of which have a slope of $1\frac{1}{4}$ to 1, then by the directions previously given

$$(16 \times 1\tfrac{1}{4} + 28) \times 16 = (20 + 28) \times 16 = 768$$

square feet, the area of the section. Then this area, multiplied by the length in feet, and the product divided by 27, will give the content of the cutting in cubic yards. For one chain of 66 feet this will be

$$\frac{768 \times 66}{27} = \frac{256 \times 22}{3} = 1877\tfrac{1}{3} \text{ cubic yards.}$$

The same process may be followed for any section long or short, which may be made to vary according to the change of the configuration of the ground.

ERRATA.

Page 3, line 3 from bottom, *for* Connaisance *read* Connaissance
— 36, line 7 from top, *for* elliptically *read* ellipticity
— 47, note, *for* log p'' and 2 *read* log $p'' \times 2$
— 58, *for* sin $\frac{1}{2}$ H, &c. see page 140

ASTRONOMICAL,

GEODETICAL, AND RAILWAY

TABLES.

Table I. Depression or Dip of the Horizon.

Ht.	Dip.	Ht.	Dip.
Feet	′ ″	Feet	′ ″
1	0 59	70	8 14
2	1 24	80	8 48
3	1 43	90	9 20
4	1 58	100	9 50
5	2 12	110	10 19
6	2 25	120	10 46
7	2 36	130	11 13
8	2 47	140	11 39
9	2 57	150	12 3
10	3 7	160	12 26
11	3 16	170	12 49
12	2 24	180	13 12
13	3 33	190	13 34
14	3 40	200	13 55
15	3 49	210	14 16
16	3 56	220	14 36
17	4 3	230	14 55
18	4 10	240	15 15
19	4 17	250	15 34
20	4 24	260	15 52
21	4 31	270	16 10
22	4 37	280	16 28
23	4 43	290	16 46
24	4 49	300	17 3
25	4 55	310	17 20
26	5 1	320	17 36
27	5 7	330	17 53
28	5 13	340	18 9
29	5 18	350	18 25
30	5 23	360	18 40
31	5 29	370	18 56
32	5 34	380	19 11
33	5 39	390	19 26
34	5 44	400	19 41
35	5 49	410	19 56
36	5 54	420	20 10
37	5 59	430	20 25
38	6 4	440	20 39
39	6 9	450	20 53
40	6 14	460	21 7
41	6 18	470	21 20
42	6 23	480	21 34
43	6 28	490	21 47
44	6 32	500	22 0
45	6 36	510	22 13
46	6 41	520	22 26
47	6 45	530	22 39
48	6 49	540	22 52
49	6 53	550	23 5
50	6 58	560	23 17
51	7 2	570	23 30
52	7 6	580	23 42
53	7 10	590	23 54
54	7 14	600	24 6
55	7 18	610	24 18
56	7 22	620	24 30
57	7 26	630	24 42
58	7 30	640	24 54
59	7 34	650	25 6
60	7 38	660	25 17

Table II. Correction of the Apparent Altitudes of the Sun and Stars.

Alt.	Star.	Sun.	Alt.	Star.	Sun.	Alt.	Star.	Sun.
° ′	′ ″	′ ″	° ′	′ ″	′ ″	°	′ ″	′ ″
0 0	34 32	34 23	10 0	5 20	5 11	30	1 41	1 33
10	32 25	32 16	20	5 10	5 1	31	1 37	1 29
20	30 33	30 24	40	5 1	4 52	32	1 33	1 26
30	28 50	28 41	11 0	4 52	4 43	33	1 30	1 22
40	27 14	27 5	20	4 44	4 35	34	1 26	1 19
50	25 47	25 38	40	4 36	4 27	35	1 23	1 16
1 0	24 27	24 18	12 0	4 28	4 19	36	1 20	1 13
10	23 13	23 4	20	4 21	4 12	37	1 17	1 10
20	22 5	21 56	40	4 14	4 5	38	1 15	1 8
30	21 3	20 54	13 0	4 8	3 59	39	1 12	1 5
40	20 5	19 56	20	4 2	3 53	40	1 10	1 2
50	19 11	19 2	40	3 56	3 47	41	1 7	1 0
2 0	18 21	18 12	14 0	3 50	3 41	42	1 5	0 59
10	17 35	17 26	20	3 45	3 36	43	1 3	0 56
20	16 52	16 43	40	3 40	3 31	44	1 0	0 54
30	16 12	16 3	15 0	3 35	3 26	45	0 58	0 52
40	15 35	15 26	20	3 30	3 21	46	0 56	0 50
50	15 0	14 51	40	3 25	3 16	47	0 54	0 48
3 0	14 27	14 18	16 0	3 21	3 12	48	0 53	0 46
10	13 56	13 47	20	3 17	3 8	49	0 51	0 45
20	13 27	13 18	40	3 13	3 4	50	0 49	0 43
30	13 0	12 51	17 0	3 9	3 0	51	0 47	0 42
40	12 34	12 25	20	3 5	2 56	52	0 46	0 40
50	12 10	12 1	40	3 1	2 52	53	0 44	0 39
4 0	11 47	11 38	18 0	2 58	2 49	54	0 42	0 37
10	11 26	11 17	20	2 54	2 45	55	0 41	0 36
20	11 6	10 56	40	2 51	2 42	56	0 39	0 34
30	10 46	10 38	19 0	2 48	2 39	57	0 38	0 33
40	10 28	10 19	20	2 45	2 37	58	0 36	0 32
50	10 11	10 2	40	2 42	2 34	59	0 35	0 31
5 0	9 54	9 45	20 0	2 39	2 31	60	0 34	0 30
10	9 38	9 29	20	2 36	2 28	61	0 32	0 28
20	9 23	9 14	40	2 34	2 25	62	0 31	0 27
30	9 9	9 0	21 0	2 31	2 22	63	0 30	0 26
40	8 55	8 47	20	2 28	2 19	64	0 28	0 25
50	8 42	8 34	40	2 26	2 17	65	0 27	0 24
6 0	8 30	8 21	22 0	2 24	2 15	66	0 26	0 23
10	8 18	8 9	20	2 21	2 13	67	0 25	0 22
20	8 7	7 58	40	2 19	2 11	68	0 24	0 21
30	7 56	7 47	23 0	2 17	2 9	69	0 22	0 19
40	7 45	7 36	20	2 15	2 7	70	0 21	0 18
50	7 35	7 26	40	2 13	2 5	71	0 20	0 17
7 0	7 25	7 17	24 0	2 10	2 3	72	0 19	0 16
10	7 16	7 7	20	2 8	2 1	73	0 18	0 15
20	7 7	6 59	40	2 7	1 59	74	0 17	0 14
30	6 59	6 50	25 0	2 5	1 57	75	0 16	0 13
40	6 50	6 42	20	2 3	1 55	76	0 15	0 12
50	6 42	6 34	40	2 1	1 54	77	0 13	0 11
8 0	6 35	6 26	26 0	1 59	1 52	78	0 12	0 10
10	6 27	6 19	20	1 57	1 50	79	0 11	0 9
20	6 20	6 11	40	1 56	1 49	80	0 10	0 8
30	6 13	6 5	27 0	1 54	1 47	81	0 9	0 8
40	6 7	5 58	20	1 53	1 45	82	0 8	0 7
50	6 0	5 51	40	1 51	1 43	83	0 7	0 6
9 0	5 54	5 45	28 0	1 49	1 41	84	0 6	0 5
10	5 48	5 39	20	1 48	1 39	85	0 5	0 4
20	5 42	5 33	40	1 46	1 38	86	0 4	0 4
30	5 36	5 28	29 0	1 45	1 36	87	0 3	0 3
40	5 31	5 22	20	1 44	1 35	88	0 2	0 2
50	5 25	5 17	40	1 42	1 34	89	0 1	0 1

Note 1. If the dip be increased by one-sixth of itself, the sum will be the distance of the visible horizon in geographical minutes and seconds.

2. If the dip be determined by observation, the height of the instrument above the sea will be found.

Table III. To correct the Mean Refraction.

Fahrenheit's Thermometer.

+	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50° +
−	90°	88°	86°	84°	82°	80°	78°	76°	74°	72°	70°	68°	66°	64°	62°	60°	58°	56°	54°	52°	50° −
Alt. °	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
4	68	65	61	58	54	51	48	44	41	37	34	31	27	24	20	17	14	10	7	3	0
5	55	52	50	47	44	41	39	36	33	30	28	25	22	19	17	14	11	8	6	3	0
6	46	44	41	39	37	35	32	30	28	25	23	21	18	16	14	12	9	7	5	2	0
7	39	37	35	33	31	29	27	25	24	22	20	18	16	14	12	10	8	6	4	2	0
8	34	32	31	29	28	26	24	22	20	19	17	15	14	12	10	8	7	5	3	2	0
10	28	26	25	23	22	21	19	18	17	15	14	12	11	10	8	7	6	4	3	1	0
12	22	21	20	19	18	17	16	14	13	12	11	10	9	8	7	6	4	3	2	1	0
15	18	17	16	15	14	13	12	11	11	10	9	8	7	6	5	4	4	3	2	1	0
18	14	14	13	12	12	11	10	9	9	8	7	7	6	5	4	4	3	2	1	1	0
22	12	11	10	10	9	9	8	8	7	6	6	5	5	4	3	3	2	2	1	1	0
30	8	8	7	7	6	6	6	5	5	4	4	4	3	3	2	2	2	1	1	0	0
50	4	4	4	3	3	3	3	3	2	2	2	2	2	1	1	1	1	1	0	0	0
60	3	3	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	0	0	0	0
70	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
80	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
−	27.16	27.30	27.44	27.59	27.73	27.87	28.01	28.15	28.30	28.44	28.58	28.72	28.86	29.01	29.15	29.29	29.43	29.57	29.72	29.86	30.00 −
+											31.42	31.28	31.14	30.99	30.85	30.71	30.57	30.43	30.28	30.14	30.00 +

Height of the English Barometer in inches and decimals.

The signs must be changed when the numbers from this Table are applied to the correction of the Moon's Altitude.

Table IV. Correction of the Apparent Altitude of the Moon.

☽'s Alt	Moon's Horizontal Parallax. 54′	55′	56′	57′	58′	59′	60′	61′	P. P. for Alt. + ′/″	0′	2′	4′	6′	8′	P. P. for Par. + 0″	2″	4″	6″	8″
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″			″	″	″	″	″	″	″	″	″
0	19 28	20 28	21 28	22 28	23 28	24 28	25 28	26 28	0						0	2	4	6	8
1	29 33	30 33	31 33	32 33	33 33	34 33	35 33	36 33	10						10	12	14	16	18
2	35 37	36 37	37 37	38 37	39 37	40 37	41 37	42 37	20						20	22	24	26	28
3	39 30	40 30	41 30	42 29	43 29	44 29	45 29	46 29	30						30	32	34	36	38
4	42 6	43 6	44 6	45 6	46 5	47 5	48 5	49 5	40						40	42	44	46	48
5	43 55	44 55	45 54	46 54	47 54	48 54	49 53	50 53	50						50	52	54	56	58
6	45 13	46 12	47 12	48 11	49 11	50 10	51 10	52 10	0						0	2	4	6	8
7	46 10	47 10	48 9	49 9	50 9	51 8	52 7	53 7	10						10	12	14	16	18
8	46 54	47 53	48 52	49 52	50 51	51 51	52 50	53 50	20						20	22	24	26	28
9	47 26	48 26	49 25	50 24	51 23	52 23	53 22	54 21	30						30	32	34	36	37
10	47 51	48 50	49 48	50 48	51 47	52 46	53 44	54 45	40						39	41	43	45	47
11	48 10	49 9	50 7	51 6	52 5	53 4	54 3	55 2	50						49	51	53	55	57
12	48 22	49 21	50 19	51 18	52 17	53 15	54 14	55 13	0	0	0	0	1	1	0	2	4	6	8
13	48 30	49 29	50 27	51 26	52 24	53 23	54 21	55 20	10	1	1	1	1	2	10	12	14	15	17
14	48 35	49 33	50 31	51 30	52 28	53 26	54 24	55 22	20	2	2	2	2	2	19	21	23	25	27
15	48 36	49 34	50 32	51 30	52 28	53 26	54 24	55 22	30	3	3	3	3	3	29	31	33	35	37
16	48 35	49 32	50 30	51 28	52 26	53 23	54 21	55 20	40	3	3	4	4	4	39	41	43	45	46
17	48 31	49 28	50 26	51 23	52 20	53 18	54 15	55 13	50	4	4	5	5	5	48	50	52	54	56
18	48 17	49 13	50 10	51 7	52 4	53 0	53 57	54 54	0	14	13	13	13	12	0	2	4	6	7
19	48 7	49 3	49 59	50 56	51 52	52 49	53 45	54 41	10	12	11	11	10	10	9	11	13	15	17
20	47 55	48 51	49 47	50 43	51 39	52 35	53 31	54 27	20	9	9	8	8	7	19	21	22	24	26
21	47 42	48 37	49 33	50 29	51 24	52 20	53 16	54 11	30	7	6	6	6	5	28	30	32	34	36
22	47 27	48 22	49 17	50 12	51 8	52 3	52 58	53 53	40	5	4	4	3	3	37	39	41	43	45
23	47 11	48 5	49 0	49 55	50 50	51 45	52 39	53 34	50	2	2	1	1	0	47	49	51	52	54
24	46 53	47 47	48 42	49 36	50 31	51 25	52 19	53 14	0	23	22	21	21	20	0	2	4	5	7
25	46 34	47 28	48 22	49 16	50 10	51 4	51 58	52 52	10	19	18	18	17	16	9	11	13	14	16
26	46 14	47 7	48 1	48 54	49 48	50 41	51 35	52 28	20	15	15	14	13	12	18	20	22	23	25
27	45 52	46 45	47 38	48 31	49 24	50 17	51 10	52 3	30	11	11	10	9	8	27	29	30	32	34
28	45 30	46 22	47 15	48 7	49 0	49 52	50 45	51 37	40	8	7	6	5	5	36	38	39	41	43
29	45 6	45 58	46 50	47 42	48 34	49 26	50 18	50 10	50	4	3	2	1	1	45	47	48	50	52

Table IV. Correction of the Apparent Altitude of the Moon.

☽'s Alt	Moon's Horizontal Parallax.								P. P. for Alt. +						P. P. for Par. +				
	54′	55′	56′	57′	58′	59′	60′	61′	″	0′	2′	4′	6′	8′	0″	2″	4″	6″	8″
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″		″	″	″	″	″	″	″	″	″	″
30	44 41	45 33	46 24	47 16	48 7	48 58	49 50	50 41	0	30	29	28	27	26	0	2	3	5	7
31	44 15	45 6	45 57	46 48	47 39	48 30	49 21	50 12	10	25	24	23	22	21	8	10	12	13	15
32	43 48	44 39	45 29	46 19	47 10	48 0	48 50	49 41	20	20	19	18	17	16	17	19	20	22	23
33	43 21	44 10	45 0	45 50	46 40	47 29	48 19	49 9	30	15	14	13	12	11	25	27	28	30	32
34	42 52	43 41	44 30	45 19	46 8	46 57	47 47	48 36	40	10	9	8	7	6	34	35	37	38	40
35	42 22	43 10	43 59	44 48	45 36	46 25	47 13	48 2	50	5	4	3	2	1	42	44	45	47	48
36	41 51	42 39	43 27	44 15	45 3	45 51	46 39	47 27	0	36	35	34	32	31	0	2	3	5	6
37	41 19	42 7	42 54	43 41	44 29	45 16	46 3	46 51	10	30	29	28	26	25	8	9	11	12	14
38	40 47	41 33	42 20	43 7	43 53	44 40	45 27	46 13	20	24	23	22	20	19	16	17	19	20	22
39	40 13	40 59	41 45	42 31	43 17	44 3	44 49	45 35	30	18	17	16	14	13	23	25	27	28	30
40	39 39	40 24	41 10	41 55	42 40	43 25	44 11	44 56	40	12	11	10	8	7	31	33	35	36	38
41	39 4	39 48	40 33	41 18	42 2	42 47	43 31	44 16	50	6	5	4	2	1	39	41	42	44	45
42	38 28	39 12	39 56	40 40	41 23	42 7	42 51	43 35	0	41	40	38	37	35	0	1	3	4	6
43	37 51	38 34	39 17	40 1	40 44	41 27	42 10	42 53	10	34	33	31	30	29	7	9	10	11	13
44	37 13	37 56	38 38	39 21	40 3	40 46	41 28	42 11	20	27	26	25	23	22	14	16	17	19	21
45	36 35	37 17	37 59	38 40	39 22	40 4	40 45	41 27	30	20	19	18	16	15	22	23	24	26	27
46	35 56	36 37	37 18	37 59	38 40	39 21	40 2	40 43	40	14	12	11	10	8	29	30	31	33	34
47	35 16	35 56	36 36	37 17	37 57	38 37	39 17	39 57	50	7	5	4	3	1	36	37	39	40	41
48	34 36	35 15	35 54	36 34	37 13	37 52	38 32	39 11	0	45	43	42	40	39	0	1	3	4	5
49	33 54	34 33	35 11	35 50	36 29	37 7	37 46	38 24	10	37	36	34	33	31	6	7	9	10	11
50	33 12	33 50	34 28	35 6	35 43	36 21	36 59	37 37	20	30	28	27	25	24	13	14	15	17	18
51	32 30	33 7	33 44	34 21	34 58	35 35	36 11	36 48	30	22	21	19	18	16	19	20	22	23	24
52	31 47	32 23	32 59	33 35	34 11	34 47	35 23	35 59	40	15	13	12	10	9	25	27	28	29	31
53	31 3	31 38	32 13	32 48	33 24	33 59	34 34	35 10	50	7	6	4	3	1	32	33	34	36	37
54	30 18	30 53	31 27	32 2	32 36	33 10	33 45	34 19	0	49	47	46	44	42	0	1	2	3	4
55	29 33	30 7	30 40	31 14	31 47	32 21	32 54	33 28	10	41	39	38	36	34	6	7	8	9	10
56	28 47	29 20	29 53	30 25	30 58	31 31	32 3	32 36	20	33	31	29	28	26	11	12	13	14	15
57	28 1	28 33	29 5	29 36	30 8	30 40	31 12	31 44	30	24	23	21	20	18	17	18	19	20	21
58	27 14	27 45	28 16	28 47	29 18	29 49	30 20	30 51	40	16	15	13	11	10	22	23	24	25	26
59	26 27	26 57	27 27	27 57	28 27	28 57	29 27	29 57	50	8	6	5	3	2	28	29	30	31	32
60	25 39	26 8	26 37	27 6	27 35	28 4	28 34	29 3	0	52	50	48	47	45	0	1	2	3	4
61	24 51	25 19	25 47	26 15	26 43	27 11	27 40	28 8	10	43	42	40	38	36	5	6	6	7	8
62	24 2	24 29	24 56	25 23	25 51	26 18	26 45	27 12	20	35	33	31	29	28	9	10	11	12	13
63	23 12	23 39	24 5	24 31	24 58	25 24	25 50	26 16	30	26	24	22	21	19	14	15	16	17	18
64	22 22	22 48	23 13	23 39	24 4	24 29	24 55	25 20	40	17	16	14	12	10	18	19	20	21	22
65	21 32	21 57	22 21	22 46	23 10	23 34	23 59	24 23	50	9	7	5	3	2	23	24	25	26	27
66	20 42	21 5	21 29	21 52	22 15	22 39	23 2	23 26	0	55	53	51	49	48	0	1	2	2	3
67	19 51	20 13	20 36	20 58	21 21	21 43	22 5	22 28	10	46	44	42	40	38	4	4	5	6	7
68	18 59	19 21	19 42	20 4	20 25	20 47	21 8	21 30	20	37	35	33	31	29	7	8	9	10	10
69	18 7	18 28	18 48	19 9	19 29	19 50	20 10	20 31	30	27	26	24	22	20	11	12	12	13	14
70	17 15	17 35	17 54	18 14	18 33	18 53	19 12	19 32	40	18	16	15	13	11	15	15	16	17	17
71	16 23	16 41	17 0	17 18	17 37	17 55	18 14	18 32	50	9	7	5	4	2	18	19	20	20	21
72	15 30	15 47	16 5	16 22	16 40	16 57	17 15	17 33	0	57	55	53	51	49	0	1	1	2	2
73	14 37	14 53	15 10	15 26	15 43	15 59	16 16	16 32	10	47	46	44	42	40	3	3	4	4	5
74	13 43	13 59	14 14	14 30	14 45	15 1	15 16	15 32	20	38	36	34	32	30	5	6	6	7	7
75	12 50	13 4	13 19	13 33	13 48	14 2	14 17	14 31	30	28	27	25	23	21	8	9	9	10	10
76	11 56	12 9	12 23	12 36	12 50	13 3	13 17	13 30	40	19	17	15	13	11	11	11	12	12	13
77	11 1	11 14	11 26	11 39	11 51	12 4	12 16	12 29	50	9	8	6	4	2	13	14	14	15	15
78	10 7	10 19	10 30	10 41	10 53	11 4	11 16	11 27	0	58	56	54	52	50	0	0	1	1	1
79	9 12	9 23	9 33	9 44	9 54	10 5	10 15	10 25	10	48	46	44	42	41	2	2	2	3	3
80	8 18	8 27	8 37	8 46	8 55	9 5	9 14	9 24	20	39	37	35	33	31	3	4	4	4	5
81	7 23	7 31	7 40	7 48	7 56	8 5	8 13	8 21	30	29	27	25	23	21	5	5	6	6	6
82	6 28	6 35	6 42	6 50	6 57	7 4	7 12	7 19	40	19	17	15	13	12	7	7	7	8	8
83	5 33	5 39	5 45	5 51	5 58	6 4	6 10	6 17	50	10	8	6	4	2	8	9	9	9	10
84	4 37	4 43	4 48	4 53	4 58	5 4	5 9	5 14	0	59	57	55	53	51	0	0	0	0	0
85	3 42	3 46	3 50	3 55	3 59	4 3	4 7	4 11	10	49	47	45	43	41	1	1	1	1	1
86	2 47	2 50	2 53	2 56	2 59	3 2	3 5	3 8	20	39	37	35	33	31	1	1	1	2	2
87	1 51	1 53	1 55	1 57	1 59	2 2	2 4	2 6	30	29	27	26	24	22	2	2	2	2	2
88	0 56	0 57	0 58	0 59	1 0	1 1	1 2	1 3	40	20	18	16	14	12	2	3	3	3	3
89	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	50	10	8	6	4	2	3	3	3	3	4
	54′	55′	56′	57′	58′	59′	60′	61′		0′	2′	4′	6′	8′	0″	2″	4″	6″	8″

Table V. Mean Refractions.

English Barometer 30 inches. Fahrenheit's Thermometer 50°.

Z. D.	Log δθ.	Diff. to 1′	Z. D.	Log δθ.	Diff. to 1′	Z. D.	Log δθ.	Diff. to 1′	$\frac{d\,\delta\theta}{d\tau}$	$\frac{d\,\delta\theta}{dp}$
°			° ′			° ′			—	
0			60 0	2.00368	29.1	80 0	2.50541	69.6	0.030	0.04
1	0.0085	50.2	20	2.00949	29.3	10	2.51237	70.7	0.031	0.04
2	0.3097	29.4	40	2.01535	29.5	20	2.51944	71.6	0.033	0.04
3	0.4860	20.8	61 0	2.02124	29.8	30	2.52660	72.7	0.034	0.04
4	0.6112	16.2	20	2.02718	30.0	40	2.53387	73.8	0.036	0.05
5	0.7086	13.3	40	2.03316	30.1	50	2.54125	74.9	0.038	0.05
6	0.7882	11.2	62 0	2.03918	30.4	81 0	2.54874	75.9	0.040	0.05
7	0.8557	9.8	20	2.04525	30.7	10	2.55635	77.2	0.042	0.06
8	0.9144	8.7	40	2.05137	30.9	20	2.56407	78.5	0.044	0.06
9	0.9663	7.7	63 0	2.05754	31.2	30	2.57192	79.7	0.046	0.07
10	1.0129	7.0	20	2.06376	31.5	40	2.57989	81.1	0.049	0.07
11	1.0552	6.4	40	2.07003	31.7	50	2.58800	82.4	0.051	0.08
12	1.0941	6.0	64 0	2.07635	32.0	82 0	2.59624	83.8	0.053	0.08
13	1.1300	5.6	20	2.08273	32.3	10	2.60462	85.1	0.057	0.09
14	1.1634	5.4	40	2.08917	32.6	20	2.61313	86.6	0.060	0.09
15	1.1947	4.9	65 0	2.09567	33.0	30	2.62179	88.3	0.063	0.10
16	1.2241	4.6	20	2.10224	33.2	40	2.63062	89.9	0.067	0.10
17	1.2519	4.4	40	2.10886	33.5	50	2.63961	91.6	0.069	0.11
18	1.2784	4.2	66 0	2.11555	33.9	83 0	2.64877	93.3	0.070	0.11
19	1.3036	4.0	20	2.12231	34.2	10	2.65810	94.8	0.074	0.12
20	1.3277	3.9	40	2.12913	34.5	20	2.66758	97.0	0.079	0.12
21	1.3507	3.7	67 0	2.13603	34.9	30	2.67728	98.5	0.085	0.13
22	1.3729	3.6	20	2.14300	35.4	40	2.68713	100.5	0.089	0.14
23	1.3945	3.4	40	2.15006	35.8	50	2.69718	102.8	0.095	0.15
24	1.4151	3.3	68 0	2.15719	36.2	84 0	2.70746	104.7	0.100	0.16
25	1.4352	3.2	20	2.16440	36.6	10	2.71793	106.9	0.107	0.17
26	1.4547	3.2	40	2.17171	37.1	20	2.72862	109.2	0.114	0.18
27	1.4736	3.1	69 0	2.17910	37.5	30	2.73954	111.6	0.122	0.19
28	1.4921	3.0	20	2.18658	38.1	40	2.75070	114.0	0.131	0.20
29	1.5102	2.9	40	2.19417	38.5	50	2.76210	116.6	0.141	0.22
30	1.5279	2.8	70 0	2.20185	39.0	85 0	2.77376	119.4	0.150	0.24
31	1.5452	2.8	20	2.20963	39.6	10	2.78570	121.9	0.161	0.25
32	1.5622	2.7	40	2.21752	40.2	20	2.79789	124.8	0.174	0.27
33	1.5790	2.7	71 0	2.22552	40.7	30	2.81037	128.0	0.189	0.30
34	1.5954	2.7	20	2.23363	41.3	40	2.82317	131.1	0.205	0.33
35	1.6116	2.6	40	2.24186	41.9	50	2.83628	134.1	0.222	0.36
36	1.6276	2.6	72 0	2.25022	42.5	86 0	2.84969	137.5	0.240	0.39
37	1.6435	2.6	20	2.25870	43.3	10	2.86344	141.3	0.260	0.43
38	1.6591	2.6	40	2.26732	44.0	20	2.87757	144.8	0.284	0.47
39	1.6746	2.6	73 0	2.27608	44.7	30	2.89205	148.8	0.310	0.51
40	1.6901	2.6	20	2.28498	45.4	40	2.90693	152.7	0.336	0.56
41	1.7055	2.5	40	2.29402	46.2	50	2.92220	157.0	0.362	0.61
42	1.7207	2.5	74 0	2.30322	47.0	87 0	2.93790	161.2	0.390	0.67
43	1.7358	2.5	20	2.31259	47.9	10	2.95402	165.8	0.430	0.75
44	1.7510	2.5	40	2.32213	48.8	20	2.97060	170.4	0.470	0.83
45	1.7661	2.5	75 0	2.33184	49.7	30	2.98764	175.8	0.520	0.91
46	1.7812	2.5	20	2.34174	50.7	40	3.00522	180.8	0.580	1.01
47	1.7964	2.5	40	2.35183	51.7	50	3.02330	186.2	0.630	1.13
48	1.8116	2.5	76 0	2.36212	52.8	88 0	3.04192	191.8	0.690	1.24
49	1.8268	2.5	20	2.37263	53.8	10	3.06110	197.7	0.780	1.41
50	1.8421	2.5	40	2.38334	55.1	20	3.08087	204.0	0.870	1.58
51	1.8575	2.6	77 0	2.39430	56.3	30	3.10127	210.2	0.960	1.75
52	1.8730	2.6	20	2.40550	57.6	40	3.12229	216.9	1.070	2.00
53	1.8886	2.6	40	2.41695	58.9	50	3.14398	223.9	1.190	2.24
54	1.9044	2.6	78 0	2.42867	60.3	89 0	3.16637	231.6	1.320	2.48
55	1.9204	2.6	20	2.44066	61.8	10	3.18943	238.8	1.520	2.91
56	1.9365	2.7	40	2.45295	63.5	20	3.21331	246.1	1.720	3.34
57	1.9529	2.7	79 0	2.46556	65.0	30	3.23792	252.9	1.920	3.77
58	1.9696	2.8	20	2.47848	66.9	40	3.26321	257.3	2.200	4.34
59	1.9865	2.8	40	2.49176	68.8	50	3.28894	275.5	2.480	5.00
60	2.0037	2.9	80 0	2.50541	69.6	90 0	3.31649		2.760	5.70

This Table contains the last edition of Ivory's Refractions, in a paper forming the Bakerian Lecture, printed in the Philosophical Transactions for 1838. The first three Tables on the opposite page correct the refractions for the English Barometer and Fahrenheit's Thermometer, as commonly used in Britain, and last two for the Metrical Barometer and Centigrade Thermometer, as employed on the Continent.

Table VI. Barometer.

P. P.	b	Log.
	In.	
+	27.0	9.95424
16	1	9.95585
32	2	9.95745
47	3	9.95904
63	4	9.96063
79	5	9.96221
95	6	9.96379
111	7	9.96536
126	8	9.96692
142	9	9.96848
	28.0	9.97004
15	1	9.97158
30	2	9.97313
46	3	9.97466
61	4	9.97620
76	5	9.97772
91	6	9.97924
106	7	9.98076
122	8	9.98227
137	9	9.98378
	29.0	9.98528
15	1	9.98677
29	2	9.98826
44	3	9.98975
59	4	9.99123
73	5	9.99270
88	6	9.99417
103	7	9.99563
118	8	9.99709
132	9	9.99855
	30.0	0.00000
14	1	0.00145
29	2	0.00289
43	3	0.00432
57	4	0.00575
71	5	0.00718
86	6	0.00860
100	7	0.01002
114	8	0.01143
129	9	0.01284
	31.0	0.01424

Table VII. Interior Thermometer.

τ	Log.	τ	Log.
°		°	
10	0.00173	50	0.00000
11	0.00169	51	9.99996
12	0.00164	52	9.99991
13	0.00160	53	9.99987
14	0.00156	54	9.99983
15	0.00151	55	9.99978
16	0.00147	56	9.99974
17	0.00143	57	9.99970
18	0.00138	58	9.99965
19	0.00134	59	9.99961
20	0.00130	60	9.99957
21	0.00126	61	9.99953
22	0.00121	62	9.99948
23	0.00117	63	9.99944
24	0.00113	64	9.99940
25	0.00108	65	9.99935
26	0.00104	66	9.99931
27	0.00100	67	9.99927
28	0.00095	68	9·99922
29	0.00091	69	9.99918
30	0.00087	70	9.99913
31	0.00083	71	9.99909
32	0.00078	72	9.99904
33	0.00074	73	9.99900
34	0.00070	74	9.99896
35	0.00065	75	9.99891
36	0.00061	76	9.99887
37	0.00057	77	9.99883
38	0.00052	78	9.99878
39	0.00048	79	9.99874
40	0.00043	80	9.99870
41	0.00039	81	9.99866
42	0.00034	82	9.99861
43	0.00030	83	9.99857
44	0.00026	84	9.99853
45	0.00021	85	9.99848
46	0.00017	86	9.99844
47	0.00013	87	9.99840
48	0.00008	88	9.99835
49	0.00004	89	9.99831
50	0.00000	90	9.99827

Table VIII. Exterior Thermometer.

P. P.	t	Log.	P. P.	t	Log.
	°			°	
—	10	0.03779	—	50	0.00000
10	1	0.03680	9	1	9.99910
20	2	0.03582	18	2	9.99820
29	3	0.03484	27	3	9.99730
39	4	0.03386	36	4	9.99640
49	5	0.03288	45	5	9.99550
59	6	0.03191	54	6	9.99460
69	7	0.03094	63	7	9.99371
78	8	0.02997	72	8	9.99282
88	9	0.02900	81	9	9.99193
	20	0.02803		60	9.99104
10	1	0.02706	9	1	9.99016
19	2	0.02609	18	2	9.98927
29	3	0.02514	26	3	9.98839
38	4	0.02418	35	4	9.98751
48	5	0.02323	44	5	9.98663
58	6	0.02227	53	6	9.98575
67	7	0.02132	62	7	9.98488
77	8	0.02037	70	8	9.98401
86	9	0.01942	79	9	9.98314
	30	0.01848		70	9.98227
9	1	0.01754	9	1	9.98140
19	2	0.01660	17	2	9.98054
28	3	0.01566	26	3	9.97967
38	4	0.01472	34	4	9.97881
47	5	0.01379	43	5	9.97795
56	6	0.01285	52	6	9.97709
66	7	0.01192	60	7	9.97623
75	8	0.01099	69	8	9.97537
85	9	0.01006	77	9	9.97452
	40	0.00914		80	9.97367
9	1	0.00822	8	1	9.97282
18	2	0.00730	17	2	9.97197
28	3	0.00638	25	3	9.97112
37	4	0.00546	34	4	9.97027
46	5	0.00455	42	5	9.96943
55	6	0.00363	50	6	9.96859
64	7	0.00272	59	7	9.96775
74	8	0.00181	67	8	9.96691
83	9	0.00090	76	9	9.96607
	50	0.00000		90	9.96524

Table IX. Metrical Barometer.

b	Log.	b	Log.
m. m.			
730	9.98137	750	9.99311
731	9.98196	751	9.99368
732	9.98256	752	9.99426
733	9.98315	753	9.99484
734	9.98374	754	9.99542
735	9.98433	755	9.99599
736	9.98492	756	9.99657
737	9.98551	757	9.99714
738	9.98610	758	9.99771
739	9.98669	759	9.99829
740	9.98728	760	9.99886
741	9.98786	761	9.99943
742	9.98845	762	0.00000
743	9.98903	763	0.00057
744	9.98962	764	0.00114
745	9.99020	765	0.00171
746	9.99078	766	0.00227
747	9.99137	767	0.00284
748	9.99195	768	0.00341
749	9.99253	769	0.00397

P. P.	1	2	3	4	5	6	7	8	9
+	6	12	17	23	29	35	41	46	52

Table X. Centigrade Thermometer.

t	Log.	t	Log.
°		°	
— 10	0.03542	+ 10	0.00000
9	0.03358	11	9.99829
8	0.03175	12	9.99659
7	0.02994	13	9.99491
6	0.02812	14	9.99322
5	0.02631	15	9.99154
4	0.02451	16	9.98987
3	0.02272	17	9.98820
2	0.02094	18	9.98654
— 1	0.01915	19	9.98488
0	0.01738	20	9.98323
+ 1	0.01563	21	9.98158
2	0.01385	22	9.97994
3	0.01210	23	9.97832
4	0.01035	24	9.97669
5	0.00861	25	9.97506
6	0.00687	26	9.97344
7	0.00515	27	9.97183
8	0.00343	28	9.97023
9	0.00171	29	9.96863

P.P.	1	2	3	4	5	6	7	8	9
—	17	34	51	68	85	102	119	136	153

Table XI. Logs to compute the Terrestrial Refraction.

Fahr Ther *t*	English Barometer, *b*.							Diff.
	24 in.	25 in.	26 in.	27 in.	28 in.	29 in.	30 in.	
°								+
30	7.45244	7.45249	7.45253	7.45258	7.45263	7.45267	7.45272	5
31	7.45239	7.45244	7.45248	7.45253	7.45258	7.45262	7.45267	5
32	7.45233	7.45238	7.45243	7.45248	7.45253	7.45258	7.45263	5
33	7.45227	7.45232	7.45238	7.45243	7.45248	7.45253	7.45259	5
34	7.45221	7.45226	7.45232	7.45237	7.45243	7.45248	7.45254	5
35	7.45215	7.45221	7.45226	7.45232	7.45238	7.45243	7.45249	6
36	7.45209	7.45215	7.45221	7.45226	7.45232	7.45238	7.45244	6
37	7.45202	7.45208	7.45214	7.45220	7.45226	7.45232	7.45238	6
38	7.45195	7.45201	7.45208	7.45214	7.45220	7.45226	7.45233	6
39	7.45188	7.45194	7.45201	7.45207	7.45214	7.45220	7.45227	7
40	7.45181	7.45188	7.45194	7.45201	7.45208	7.45214	7.45221	7
41	7.45173	7.45180	7.45187	7.45194	7.45201	7.45208	7.45215	7
42	7.45165	7.45172	7.45180	7.45187	7.45194	7.45201	7.45209	7
43	7.45157	7.45164	7.45172	7.45179	7.45187	7.45194	7.45202	7
44	7.45148	7.45156	7.45164	7.45171	7.45179	7.45187	7.45195	8
45	7.45140	7.45148	7.45156	7.45164	7.45172	7.45180	7.45188	8
46	7.45131	7.45139	7.45148	7.45156	7.45164	7.45172	7.45181	8
47	7.45121	7.45130	7.45139	7.45147	7.45156	7.45165	7.45174	9
48	7.45111	7.45120	7.45129	7.45139	7.45148	7.45157	7.45166	9
49	7.45101	7.45110	7.45120	7.45129	7.45139	7.45148	7.45158	9
50	7.45091	7.45101	7.45111	7.45120	7.45130	7.45140	7.45150	10
51	7.45080	7.45090	7.45100	7.45110	7.45121	7.45131	7.45141	10
52	7.45069	7.45079	7.45090	7.45100	7.45111	7.45121	7.45132	10
53	7.45058	7.45069	7.45080	7.45090	7.45101	7.45112	7.45123	11
54	7.45046	7.45057	7.45068	7.45080	7.45091	7.45102	7.45113	11
55	7.45034	7.45046	7.45057	7.45069	7.45081	7.45092	7.45104	12
56	7.45021	7.45033	7.45045	7.45058	7.45070	7.45082	7.45094	12
57	7.45008	7.45021	7.45033	7.45046	7.45059	7.45071	7.45084	13
58	7.44994	7.45007	7.45020	7.45034	7.45047	7.45060	7.45073	13
59	7.44981	7.44994	7.45008	7.45021	7.45035	7.45048	7.45062	14
60	7.44967	7.44981	7.44995	7.45009	7.45023	7.45037	7.45051	14
61	7.44952	7.44966	7.44981	7.44995	7.45010	7.45024	7.45039	15
62	7.44937	7.44952	7.44967	7.44982	7.44997	7.45012	7.45027	15
63	7.44921	7.44936	7.44952	7.44967	7.44983	7.44998	7.45014	16
64	7.44905	7.44921	7.44937	7.44953	7.44969	7.44985	7.45001	16
65	7.44889	7.44905	7.44922	7.44938	7.44955	7.44971	7.44988	17
66	7.44872	7.44889	7.44906	7.44922	7.44939	7.44956	7.44973	17
67	7.44854	7.44871	7.44889	7.44906	7.44923	7.44941	7.44958	17
68	7.44835	7.44853	7.44871	7.44890	7.44908	7.44926	7.44944	18
69	7.44817	7.44836	7.44855	7.44873	7.44892	7.44911	7.44930	18
70	7.44798	7.44817	7.44837	7.44856	7.44876	7.44895	7.44915	19
71	7.44778	7.44798	7.44818	7.44839	7.44859	7.44879	7.44899	20
72	7.44757	7.44778	7.44799	7.44820	7.44841	7.44862	7.44883	21
73	7.44735	7.44757	7.44779	7.44800	7.44822	7.44844	7.44866	22
74	7.44713	7.44735	7.44758	7.44780	7.44803	7.44825	7.44848	23
75	7.44691	7.44714	7.44737	7.44761	7.44784	7.44807	7.44830	24
76	7.44668	7.44692	7.44716	7.44740	7.44763	7.44787	7.44811	24
77	7.44644	7.44669	7.44693	7.44718	7.44742	7.44767	7.44792	25
78	7.44619	7.44645	7.44670	7.44695	7.44721	7.44746	7.44772	26
79	7.44593	7.44619	7.44646	7.44672	7.44698	7.44724	7.44751	26
80	7.44567	7.44594	7.44622	7.44649	7.44676	7.44704	7.44731	27
81	7.44542	7.44570	7.44598	7.44626	7.44654	7.44682	7.44710	28
82	7.44515	7.44544	7.44573	7.44601	7.44630	7.44659	7.44688	29
83	7.44486	7.44516	7.44546	7.44575	7.44605	7.44635	7.44665	30
84	7.44455	7.44486	7.44517	7.44549	7.44580	7.44611	7.44642	31
85	7.44424	7.44456	7.44489	7.44521	7.44553	7.44586	7.44618	32
86	7.44393	7.44426	7.44460	7.44493	7.44526	7.44560	7.44593	33
87	7.44361	7.44395	7.44430	7.44464	7.44498	7.44533	7.44567	34
88	7.44328	7.44363	7.44399	7.44434	7.44470	7.44506	7.44541	35
89	7.44294	7.44331	7.44367	7.44404	7.44441	7.44477	7.44514	36
90	7.44258	7.44296	7.44334	7.44372	7.44410	7.44448	7.44486	38
°								°
30	— 6	— 6	— 5	— 5	— 5	— 5	— 5	30
40	8	8	7	7	7	6	6	40
50	10	10	10	9	9	8	8	50
60	14	14	14	14	13	12	11	60
70	19	19	18	18	17	16	15	70
80	26	26	24	23	22	21	20	80
90	35	35	33	32	30	29	28	90

Table XII. Parallax of the Sun in Altitude or Z. D.

Alt.	Jan. 1	Feb. Dec. 1	Mar. Nov. 1	Apr. Oct. 1	May Sept. 1	June Aug. 1	July 1
°	″	″	″	″	″	″	″
0	8.8	8.7	8.7	8.6	8.5	8.5	8.5
3	8.7	8.7	8.7	8.6	8.5	8.5	8.4
6	8.7	8.7	8.6	8.5	8.5	8.4	8.4
9	8.6	8.6	8.6	8.5	8.4	8.4	8.3
12	8.5	8.5	8.5	8.4	8.3	8.3	8.3
15	8.4	8.4	8.4	8.3	8.2	8.2	8.2
18	8.3	8.3	8.2	8.2	8.1	8.1	8.0
21	8.2	8.2	8.1	8.0	8.0	7.9	7.9
24	8.0	8.0	7.9	7.8	7.8	7.7	7.7
27	7.8	7.8	7.7	7.7	7.6	7.5	7.5
30	7.6	7.6	7.5	7.4	7.4	7.3	7.3
33	7.3	7·3	7.3	7.2	7.1	7.1	7.1
36	7.1	7.1	7.0	7.0	6.9	6.9	6.8
39	6.8	6.8	6.7	6.7	6.6	6.6	6.6
42	6.5	6.5	6.4	6.4	6.3	6.3	6.3
45	6.2	6.2	6.1	6.1	6.0	6.0	6.0
48	5.8	5.8	5.8	5.7	5.7	5.7	5.7
51	5.5	5.5	5.5	5.4	5.4	5.3	5.3
54	5.1	5.1	5.1	5.0	5.0	5.0	5.0
57	4.8	4.7	4.7	4.7	4.6	4.6	4.6
60	4.4	4.4	4.3	4.3	4.2	4.2	4.2
63	4.0	4.0	3.9	3.9	3.9	3.8	3.8
66	3.6	3.5	3.5	3.5	3.5	3.4	3.4
69	3.1	3.1	3.1	3.1	3.1.	3.0	3.0
72	2.7	2.7	2.7	2.7	2.6	2.6	2.6
75	2.3	2.3	2.2	2.2	2.2	2.2	2.2
78	1.8	1.8	1.8	1.8	1.8	1.8	1.8
81	1.4	1.4	1.4	1.3	1.3	1.3	1.3
84	0.9	0.9	0.9	0.9	0.9	0.9	0.9
87	0.5	0.5	0.5	0.4	0.4	0.4	0.4
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table XIII. Parallax of the Planets in Altitude or Zenith-Distance.

Horizontal Parallax.												Z.D.
10″	20″	30″	1″	2″	3″	4″	5″	6″	7″	8″	9″	
″	″	″	″	″	″	″	″	″	″	″	″	°
10.0	20.0	30.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	90
10.0	20.0	30.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	87
9.9	19.9	29.8	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	84
9.9	19.8	29.6	1.0	2.0	3.0	4.0	4.9	5.9	6.9	7.9	8.9	81
9.8	19.6	29.3	1.0	2.0	2.9	3.9	4.9	5.9	6.8	7.8	8.8	78
9.7	19.3	29.0	1.0	1.9	2.9	3.9	4.8	5.8	6.8	7.7	8.7	75
9.5	19.0	28.5	1.0	1.9	2.9	3.8	4.8	5.7	6.7	7.6	8.6	72
9.3	18.7	28.0	0.9	1.9	2.8	3.7	4.7	5.6	6.5	7.5	8.4	69
9.1	18.3	27.4	0.9	1.8	2.7	3.7	4.6	5.5	6.4	7.3	8.2	66
8.9	17.8	26.7	0.9	1.8	2.7	3.6	4.5	5.3	6.2	7.1	8.0	63
8.7	17.3	26.0	0.9	1.7	2.6	3.5	4.3	5.2	6.1	6.9	7.8	60
8.4	16.8	25.2	0.8	1.7	2.5	3.4	4.2	5.0	5.9	6.7	7.5	57
8.1	16.2	24.3	0.8	1.6	2.4	3.2	4.0	4.9	5.7	6.5	7.3	54
7.8	15.5	23.3	0.8	1.6	2.3	3.1	3.9	4.7	5.4	6.2	7.0	51
7.4	14.9	22.3	0.7	1.5	2.2	3.0	3.7	4.5	5.2	5.9	6.7	48
7.1	14.1	21.2	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.7	6.4	45
6.7	13.4	20.1	0.7	1.3	2.0	2.7	3.3	4.0	4.7	5.4	6.0	42
6.3	12.6	18.9	0.6	1.3	1.9	2.5	3.1	3.8	4.4	5.0	5.7	39
5.9	11.8	17.6	0.6	1.2	1.8	2.4	2.9	3.5	4.1	4.7	5.3	36
5.4	10.9	16.3	0.5	1.1	1.6	2.2	2.7	3.3	3.8	4.4	4.9	33
5.0	10.0	15.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	30
4.5	9.1	13.6	0.5	0.9	1.4	1.8	2.3	2.7	3.2	3.6	4.1	27
4.1	8.1	12.2	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.7	24
3.6	7.2	10.8	0.4	0.7	1.1	1.4	1.8	2.2	2.5	2.9	3.2	21
3.1	6.2	9.3	0.3	0.6	0.9	1.2	1.5	1.9	2.2	2.5	2.8	18
2.6	5.2	7.8	0.3	0.5	0.8	1.0	1.3	1.6	1.8	2.1	2.3	15
2.1	4.2	6.2	0.2	0.4	0.6	0.8	1.0	1.2	1.5	1.7	1.9	12
1.6	3.1	4.7	0.2	0.3	0.5	0.6	0.8	0.9	1.1	1.3	1.4	9
1.0	2.1	3.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	6
0.5	1.0	1.6	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	0.5	3
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0

Table XIV. Augmentation of the Moon's semidiameter in Altitude or Z. D. +

Alt.	14′ 30″	15′ 0″	15′ 30″	16′ 0″	16′ 30″	17′ 0″	Z.D.
°	″	″	″	″	″	″	°
0	0.1	0.1	0.1	0.1	0.1	0.2	90
3	0.8	0.9	0.9	1.0	1.1	1.1	87
6	1.5	1.6	1.7	1.8	2.0	2.1	84
9	2.2	2.4	2.5	2.7	2.9	3.1	81
12	2.9	3.1	3.4	3.5	3.8	4.0	78
15	3.6	3.9	4.1	4.4	4.7	5.0	75
18	4.3	4.6	4.9	5.2	5.5	5.9	72
21	4.9	5.3	5.6	6.0	6.4	6.8	69
24	5.6	6.0	6.4	6.8	7.3	7.7	66
27	6.2	6.7	7.1	7.6	8.1	8.6	63
30	6.9	7.3	7.8	8.3	8.9	9.4	60
33	7.5	8.0	8.5	9.1	9.7	10.3	57
36	8.0	8.6	9.2	9.8	10.4	11.1	54
39	8.6	9.2	9.8	10.5	11.2	11.8	51
42	9.1	9.8	10.5	11.2	11.9	12.6	48
45	9.7	10.3	11.1	11.8	12.5	13.3	45
48	10.2	10.9	11.6	12.4	13.2	14.0	42
51	10.6	11.4	12.1	12.9	13.8	14.6	39
54	11.1	11.8	12.6	13.5	14.3	15.2	36
57	11.5	12.3	13.1	14.0	14.9	15.8	33
60	11.8	12.7	13.5	14.4	15.3	16.3	30
63	12.2	13.0	13.9	14.8	15.8	16.8	27
66	12.5	13.4	14.3	15.2	16.2	17.2	24
69	12.8	13.6	14.6	15.5	16.5	17.6	21
72	13.0	13.9	14.9	15.8	16.8	17.9	18
75	13.2	14.1	15.1	16.1	17.1	18.2	15
78	13.4	14.3	15.3	16.3	17.3	18.4	12
81	13.5	14.4	15.4	16.4	17.5	18.6	9
84	13.6	14.5	15.5	16.5	17.6	18.7	6
87	13.6	14.6	15.6	16.6	17.7	18.8	3
90	13.7	14.6	15.6	16.7	17.7	18.8	0

Table XV. Reduction of the Moon's Parallax in the Spheroid —

Lat	54′	55′	56′	57′	58′	59′	60′	61′
°	″	″	″	″	″	″	″	″
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2
9	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
12	0.4	0.5	0.5	0.5	0.5	0.5	0.5	0.6
15	0.8	0.8	0.8	0.8	0.8	0.8	0.9	0.9
18	1.1	1.1	1.1	1.1	1.2	1.2	1.2	1.2
21	1.4	1.5	1.5	1.5	1.5	1.5	1.5	1.6
24	1.8	1.8	1.9	1.9	1.9	1.9	2.0	2.0
27	2.2	2.3	2.3	2.4	2.4	2.4	2.4	2.5
30	2.7	2.7	2.8	2.8	2.9	2.9	3.0	3.1
33	3.2	3.3	3.3	3.4	3.4	3.5	3.5	3.6
36	3.7	3.8	3.8	3.9	4.0	4.0	4.0	4.1
39	4.3	4.3	4.4	4.5	4.5	4.6	4.7	4.7
42	4.9	4.9	4.9	5.0	5.1	5.2	5.3	5.4
45	5.4	5.4	5.5	5.5	5.6	5.8	5.9	6.0
48	6.0	6.0	6.1	6.2	6.3	6.4	6.5	6.6
51	6.5	6.5	6.6	6.7	6.8	6.9	7.1	7.2
54	7.0	7.1	7.2	7.3	7.4	7.6	7.7	7.8
57	7.5	7.6	7.7	7.8	7.9	8.1	8.2	8.4
60	8.0	8.1	8.2	8.4	8.6	8.7	8.8	9.0
63	8.5	8.5	8.6	8.8	9.0	9.2	9.4	9.5
66	9.0	9.0	9.2	9.3	9.5	9.7	9.8	10.0
69	9.4	9.4	9.6	9.7	9.9	10.2	10.3	10.5
72	9.7	9.8	10.1	10.2	10.3	10.5	10.8	10.9
75	10.0	10.2	10.3	10.4	10.6	10.8	11.1	11.4
78	10.2	10.3	10.5	10.7	10.9	11.2	11.4	11.6
81	10.4	10.5	10.7	10.9	11.1	11.4	11.6	11.7
84	10.6	10.8	11.0	11.2	11.4	11.6	11.7	11.9
87	10.7	10.8	11.2	11.3	11.5	11.7	11.8	12.1
90	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table XVI. Reduction of the Lat.

Lat	Reduction
°	′ ″
0	0 0.0
3	1 11.8
6	2 22.7
9	3 32.1
12	4 39.3
15	5 43.4
18	6 43.7
21	7 39.7
24	8 30.7
27	9 16.1
30	9 55.4
33	10 28.3
36	10 54.3
39	11 13.2
42	11 24.7
45	11 28.7
48	11 25.2
51	11 14.1
54	10 55.7
57	10 30.0
60	9 57.4
63	9 18.3
66	8 32.9
69	7 42.0
72	6 45.9
75	5 45.4
78	4 41.0
81	3 33.5
84	2 23.7
87	1 12.3

TABLE XVII. Reduction to the Meridian. Versines.

Time from the Meridian.

s	0 m.		1 m.		2 m.		3 m.		4 m.		5 m.		6 m.		7 m.	
	V	v	V	v	V	v	V	v	V	v	V	v	V	v	V	v
0	0	0	95	0	381	1	857	7	1523	23	2380	57	3427	117	4664	218
1	0	0	98	0	387	1	866	7	1536	24	2396	57	3446	118	4686	220
2	0	0	101	0	394	1	876	8	1548	24	2412	58	3465	120	4708	222
3	0	0	105	0	400	1	885	8	1561	24	2428	59	3484	121	4731	224
4	0	0	108	0	407	1	895	8	1574	25	2444	60	3503	123	4753	226
5	0	0	111	0	413	1	905	8	1587	25	2460	61	3522	124	4775	228
6	1	0	115	0	420	2	915	8	1600	26	2476	61	3542	125	4798	230
7	1	0	118	0	426	2	925	9	1613	26	2492	62	3561	127	4820	232
8	2	0	122	0	433	2	935	9	1626	26	2508	63	3581	128	4843	235
9	2	0	126	0	440	2	945	9	1639	27	2524	64	3600	130	4866	237
10	3	0	130	0	447	2	955	9	1652	27	2541	65	3620	131	4888	239
11	3	0	133	0	454	2	965	9	1666	28	2557	65	3639	132	4911	241
12	4	0	137	0	461	2	975	10	1679	28	2574	66	3659	133	4934	243
13	5	0	141	0	468	2	985	10	1692	29	2590	67	3679	135	4957	246
14	6	0	145	0	475	2	995	10	1706	29	2607	68	3698	137	4980	248
15	6	0	149	0	482	2	1006	10	1719	30	2623	69	3718	138	5003	250
16	7	0	153	0	489	3	1016	10	1733	30	2640	70	3738	140	5026	253
17	8	0	157	0	496	3	1026	11	1746	30	2657	71	3758	141	5049	255
18	9	0	161	0	503	3	1037	11	1760	31	2674	71	3778	143	5072	257
19	10	0	165	0	510	3	1047	11	1773	31	2691	72	3798	144	5096	260
20	11	0	169	0	518	3	1058	11	1787	32	2708	73	3818	146	5119	262
21	12	0	174	0	525	3	1068	12	1801	32	2725	74	3838	147	5142	264
22	13	0	178	0	533	3	1079	12	1815	33	2742	75	3858	149	5165	267
23	14	0	183	0	540	3	1089	12	1829	33	2759	76	3879	150	5189	269
24	15	0	187	0	548	3	1100	12	1843	34	2776	77	3899	152	5212	272
25	16	0	192	0	556	3	1111	12	1857	34	2793	78	3919	154	5236	274
26	18	0	196	0	563	4	1122	13	1871	35	2810	79	3939	155	5259	277
27	19	0	201	0	571	4	1133	13	1885	35	2827	80	3960	157	5283	279
28	21	0	205	0	579	4	1144	13	1899	36	2844	81	3980	158	5307	282
29	22	0	210	0	587	4	1155	13	1913	36	2861	82	4000	160	5331	284
30	24	0	214	0	595	4	1166	14	1927	37	2879	83	4021	162	5354	287
31	25	0	219	0	603	4	1177	14	1942	38	2896	84	4042	163	5378	289
32	27	0	224	0	611	4	1188	14	1956	38	2914	85	4063	165	5402	291
33	29	0	229	0	619	4	1200	14	1970	39	2932	86	4084	167	5426	294
34	31	0	234	0	627	4	1211	15	1985	39	2949	87	4104	168	5450	297
35	32	0	239	0	635	4	1222	15	1999	40	2967	88	4125	170	5474	300
36	34	0	244	0	643	5	1234	15	2014	40	2985	89	4146	172	5498	302
37	36	0	249	0	651	5	1245	16	2028	41	3003	90	4167	174	5522	305
38	38	0	254	0	660	5	1257	16	2043	42	3021	91	4188	175	5546	308
39	40	0	259	0	668	5	1268	16	2058	42	3039	92	4209	177	5571	310
40	42	0	264	1	677	5	1280	16	2073	43	3057	93	4230	179	5595	313
41	44	0	269	1	685	5	1292	17	2088	44	3075	94	4251	181	5619	316
42	46	0	275	1	694	5	1303	17	2103	44	3093	96	4273	183	5643	318
43	49	0	280	1	703	5	1315	17	2118	45	3111	97	4294	184	5668	321
44	51	0	286	1	711	5	1327	18	2133	45	3129	98	4316	186	5692	324
45	53	0	291	1	719	5	1339	18	2148	46	3147	99	4337	188	5717	327
46	56	0	297	1	728	6	1351	18	2163	47	3165	100	4358	190	5741	330
47	58	0	302	1	737	6	1363	19	2178	47	3184	101	4380	192	5766	332
48	61	0	308	1	746	6	1375	19	2193	48	3202	103	4401	194	5791	335
49	64	0	314	1	755	6	1387	19	2208	48	3220	104	4423	196	5816	338
50	67	0	320	1	764	6	1399	20	2224	49	3239	105	4444	197	5840	341
51	69	0	326	1	773	6	1411	20	2239	50	3257	106	4466	199	5865	344
52	72	0	332	1	782	6	1423	20	2255	51	3276	107	4488	201	5890	347
53	75	0	338	1	791	6	1435	21	2270	52	3295	109	4510	203	5915	350
54	78	0	344	1	800	6	1448	21	2286	52	3313	110	4532	205	5940	353
55	80	0	350	1	810	6	1460	21	2301	53	3332	111	4554	207	5966	356
56	83	0	356	1	819	7	1473	22	2317	54	3351	112	4576	209	5991	359
57	86	0	362	1	828	7	1485	22	2333	54	3370	114	4598	211	6016	362
58	89	0	369	1	838	7	1498	22	2348	55	3389	115	4620	213	6041	365
59	92	0	375	1	847	7	1510	23	2364	56	3408	116	4642	215	6067	368
s																
0.1	0		0		1		1		1		2		2		2	
0.2	0		1		2		2		3		4		4		5	
0.3	1		1		2		3		4		5		6		7	
0.4	1		2		3		4		6		7		8		10	
0.5	1		2		4		5		7		9		10		12	
0.6	1		3		5		7		8		11		13		14	
0.7	1		3		6		8		10		13		15		17	
0.8	1		4		6		9		11		14		17		19	
0.9	2		4		7		10		13		16		19		22	

Table XVII. Reduction to the Meridian. Versines.

Time from the Meridian.

s	8 m. V	8 m. v	9 m. V	9 m. v	10 m. V	10 m. v	11 m. V	11 m. v	12 m. V	12 m. v	13 m. V	13 m. v
0	6092	371	7710	594	9518	906	11516	1326	13705	1878	16083	2587
1	6117	374	7738	599	9550	912	11551	1334	13743	1889	16124	2600
2	6143	377	7767	603	9581	918	11586	1342	13781	1899	16166	2613
3	6168	380	7795	608	9613	924	11621	1350	13819	1910	16207	2627
4	6194	384	7824	612	9645	930	11656	1359	13857	1920	16249	2640
5	6219	387	7853	617	9677	936	11691	1367	13895	1931	16290	2654
6	6245	390	7882	621	9709	942	11726	1375	13934	1942	16332	2667
7	6270	393	7911	626	9741	949	11762	1383	13972	1953	16373	2681
8	6296	396	7940	630	9773	955	11797	1392	14011	1963	16415	2695
9	6322	400	7969	635	9805	961	11832	1400	14049	1974	16457	2708
10	6348	403	7998	640	9837	968	11868	1408	14088	1984	16498	2722
11	6374	406	8027	644	9870	974	11903	1417	14126	1995	16540	2736
12	6400	410	8056	649	9902	980	11939	1425	14165	2006	16582	2750
13	6426	413	8085	654	9934	987	11974	1434	14204	2017	16624	2764
14	6452	416	8114	658	9967	993	12010	1442	14242	2028	16666	2778
15	6479	420	8144	663	9999	1000	12045	1451	14281	2039	16708	2792
16	6505	423	8173	668	10032	1006	12081	1460	14320	2051	16750	2806
17	6531	427	8202	673	10065	1013	12117	1468	14359	2062	16792	2820
18	6557	430	8232	678	10097	1020	12153	1477	14398	2073	16834	2834
19	6584	433	8261	682	10130	1026	12189	1486	14437	2084	16876	2848
20	6610	437	8291	687	10163	1033	12225	1495	14476	2095	16918	2862
21	6636	440	8321	692	10196	1039	12261	1503	14515	2107	16960	2876
22	6663	444	8350	697	10228	1046	12297	1512	14554	2118	17003	2891
23	6689	447	8380	702	10261	1053	12333	1521	14594	2130	17045	2905
24	6716	451	8410	707	10294	1060	12369	1530	14633	2141	17088	2920
25	6743	455	8440	712	10327	1066	12405	1539	14672	2153	17130	2934
26	6769	458	8470	717	10360	1073	12441	1548	14712	2164	17173	2949
27	6796	462	8500	722	10394	1080	12478	1557	14751	2176	17215	2964
28	6823	466	8530	728	10427	1087	12514	1566	14791	2187	17258	2978
29	6850	469	8560	733	10460	1094	12550	1575	14831	2199	17301	2993
30	6877	473	8590	738	10493	1101	12587	1584	14870	2211	17344	3008
31	6904	477	8620	743	10527	1108	12623	1593	14910	2223	17387	3023
32	6931	480	8650	748	10560	1115	12660	1603	14950	2235	17430	3038
33	6958	484	8680	753	10593	1122	12696	1612	14990	2247	17473	3053
34	6985	488	8711	758	10627	1129	12733	1621	15029	2259	17516	3068
35	7013	492	8741	764	10660	1136	12769	1630	15069	2271	17559	3083
36	7040	496	8772	769	10694	1144	12806	1640	15109	2283	17602	3098
37	7067	499	8802	775	10728	1151	12843	1649	15149	2295	17645	3113
38	7094	503	8833	780	10761	1158	12880	1659	15189	2307	17688	3129
39	7122	507	8863	786	10795	1165	12917	1668	15229	2319	17732	3144
40	7149	511	8894	791	10829	1172	12954	1678	15269	2331		
41	7177	515	8925	797	10863	1180	12991	1687	15309	2344		
42	7204	519	8955	802	10897	1187	13028	1697	15349	2356		
43	7232	523	8986	807	10931	1195	13065	1707	15390	2369		
44	7260	527	9017	813	10965	1203	13102	1717	15430	2381		
45	7288	531	9048	819	10999	1210	13139	1726	15470	2393		
46	7315	535	9079	824	11033	1217	13177	1736	15511	2406		
47	7343	539	9110	830	11067	1225	13214	1746	15551	2418		
48	7371	543	9141	836	11101	1232	13252	1756	15592	2431		
49	7399	547	9172	841	11135	1240	13289	1766	15633	2444		
50	7427	552	9203	847	11170	1248	13327	1776	15673	2456		
51	7455	556	9235	853	11204	1255	13364	1786	15714	2469		
52	7483	560	9266	859	11239	1263	13402	1796	15755	2482		
53	7511	564	9297	864	11273	1271	13440	1806	15796	2495		
54	7539	568	9328	870	11308	1279	13477	1816	15837	2508		
55	7568	573	9360	876	11342	1286	13515	1826	15878	2521		
56	7596	577	9391	882	11377	1294	13553	1837	15919	2534		
57	7624	581	9423	888	11412	1302	13591	1847	15960	2547		
58	7653	586	9454	894	11446	1310	13629	1857	16001	2560		
59	7681	590	9486	900	11481	1318	13667	1861	16042	2573		
s												
0.1	3	0	3	0	3	1	4	1	4	1		
0.2	5	1	6	1	7	1	7	2	8	2		
0.3	8	1	9	1	10	2	11	3	12	4		
0.4	11	2	12	2	13	3	15	4	16	5		
0.5	13	2	15	2	16	3	18	4	20	6		
0.6	16	2	18	3	20	4	22	5	24	7		
0.7	19	3	21	3	23	5	26	6	28	8		
0.8	22	3	24	4	26	6	29	7	32	10		
0.9	24	4	27	4	29	6	33	8	36	11		

Logarithms for V′ +

No.	Logs.
1	8.314425
2	8.013395
4	7.712365
6	7.536274
8	7.411335
10	7.314425
12	7.235244
14	7.168297
16	7.110305
18	7.059152
20	7.013395

Logarithms for v′ +

No.	Logs.
1	6.013395
2	5.712365
4	5.411335
6	5.235244
8	5.110305
10	5.013395
12	4.934214
14	4.867267
16	4.809275
18	4.758122
20	4.712365

TABLE XVIII. To compute the Equation to Equal Altitudes and Equal Azimuths.

E. T.	Log A.	Log B	Log C	E. T.	Log A	Log B	Log C
h. m.				h. m.			
2 0	+9.4109	+9.3958	0.5870	13 0	+9.6405	—8.7562	0.8166
10	4117	3940	5879	10	6474	8.8296	8235
20	4127	3921	5888	20	6544	8.8941	8306
30	4137	3900	5898	30	6616	8.9518	8378
40	4148	3877	5909	40	6689	9.0043	8451
50	4159	3853	5920	50	6764	9.0524	8525
3 0	+9.4171	+9.3827	0.5933	14 0	+9.6840	—9.0970	0.8601
10	4184	3800	5945	10	6918	1387	8679
20	4198	3770	5959	20	6998	1779	8759
30	4212	3739	5973	30	7079	2150	8840
40	4227	3706	5988	40	7162	2502	8923
50	4243	3671	6004	50	7247	2839	9008
4 0	+9.4259	+9.3635	0.6021	15 0	+9.7333	—9.3162	0.9094
10	4276	3596	6038	10	7422	3472	9182
20	4294	3555	6056	20	7512	3771	9273
30	4313	3512	6074	30	7604	4061	9366
40	4333	3466	6094	40	7699	4343	9460
50	4353	3416	6114	50	7795	4617	9557
5 0	+9.4374	+9.3368	0.6135	16 0	+9.7894	—9.4884	0.9656
10	4396	3316	6155	10	7995	5145	9757
20	4418	3260	6179	20	8099	5401	9860
30	4441	3202	6202	30	8205	5652	0.9966
40	4465	3141	6226	40	8313	5899	1.0075
50	4490	3077	6251	50	8424	6142	1.0186
6 0	+9.4515	+9.3010	0.6276	17 0	+9.8538	—9.6382	1.0300
10	4541	2939	6303	10	8655	6620	0416
20	4568	2865	6330	20	8775	6855	0536
30	4596	2787	6358	30	8898	7089	0659
40	4625	2705	6386	40	9024	7320	0785
50	4654	2620	6416	50	9153	7551	0915
7 0	+9.4685	+9.2529	0.6446	18 0	+9.9286	—9.7781	1.1048
10	4716	2434	6477	10	9423	8011	1184
20	4748	2334	6509	20	9564	8240	1325
30	4781	2228	6542	30	9709	8470	1470
40	4814	2116	6575	40	9.9858	8701	1620
50	4849	1998	6610	50	0.0012	8933	1774
8 0	+9.4884	+9.1874	0.6645	19 0	+0.0171	—9.9166	1.1933
10	4920	1742	6682	10	0336	9401	2097
20	4957	1601	6719	20	0506	9639	2267
30	4995	1452	6757	30	0681	—9.9880	2443
40	5034	1294	6795	40	0864	—0.0124	2625
50	5074	1124	6834	50	1053	—0.0372	2814
9 0	+9.5114	+9.0943	0.6876	20 0	+0.1249	—0.0624	1.3010
10	5156	0749	6918	10	1453	0882	3215
20	5199	0540	6960	20	1666	1146	3428
30	5242	0313	7004	30	1889	1416	3650
40	5287	9.0068	7048	40	2122	1694	3883
50	5332	8.9801	7094	50	2366	1981	4127
10 0	+9.5379	+8.9509	0.7140	21 0	+0.2622	—0.2278	1.4383
10	5426	9186	7188	10	2893	2587	4654
20	5475	8828	7236	20	3178	2908	4940
30	5525	8427	7286	30	3482	3245	5243
40	5575	7972	7336	40	3805	3600	5566
50	5627	7449	7388	50	4151	3974	5912
11 0	+9.5680	+8.6837	0.7441	22 0	+0.4523	—0.4372	1.6284
10	5734	6102	7495	10	4925	4799	6687
20	5789	5191	7550	20	5365	5260	7125
30	5845	4001	7606	30	5848	5764	7609
40	5902	8.2299	7663	40	6386	6319	8147
50	5960	+7.9348	7722	50	6992	6941	8753
12 0	+9.6020		0.7782	23 0	+0.7689	—0.7651	1.9450
10	6081	—7.9469	7842	10	0.8508	8482	2.0269
20	6143	8.2540	7905	20	0.9505	9489	1267
30	6207	4363	7968	30	1.0783	—1.0774	2544
40	6272	5675	8033	40	1.2573	1.2569	4334
50	6338	6707	8099	50	1.5613	1.5612	7374

Table XIX. To convert feet on the Terrestrial Spheroid into seconds of Arc, and conversely.

Lat.	Log M. 0° 360°	Diff.	Azimuth from the Meridian, or Z, Log O. 10° 350°	20° 340°	30° 330°	40° 320°	50° 310°	60° 300°	70° 290°	80° 280°	Log P. 90° 270°	D.
0°	7.9967088	13	66216	63706	59856	55129	50092	45353	41488	38965	38086	5
1	67075	40	66203	63693	59844	55120	50085	45347	41481	38960	38081	13
2	67035	66	66166	63656	59812	55091	50059	45327	41466	38945	38068	22
3	66969	93	66101	63595	59755	55042	50020	45295	41438	38922	38046	31
4	66876	118	66008	63510	59679	54974	49963	45248	41402	38889	38015	39
5	66758	144	65893	63400	59579	54889	49892	45185	41353	38848	37976	48
6	66614	170	65753	63268	59460	54789	49803	45117	41293	38796	37928	57
7	66444	196	65585	63111	59318	54662	49700	45031	41223	38736	37871	65
8	66248	221	65394	62930	59156	54519	49581	44934	41143	38667	37806	74
9	66027	244	65177	62727	58970	54359	49445	44821	41051	38588	37732	81
10	7.9965783	270	64938	62502	58767	54182	49296	44701	40960	38503	37651	90
11	65513	297	64675	62251	58541	53985	49133	44564	40839	38407	37561	99
12	65216	321	64382	61978	58294	53770	48952	44416	40718	38303	37462	107
13	64895	345	64068	61682	58026	53539	48756	44255	40586	38188	37355	115
14	64550	368	63730	61363	57738	53286	48545	44083	40444	38066	37240	122
15	64182	391	63369	61024	57432	53021	48321	43900	40293	37937	37118	130
16	63791	411	62985	60663	57105	52736	48082	43704	40131	37800	36988	138
17	63380	432	62582	60283	56762	52438	47832	43498	39961	37653	36850	144
18	62948	456	62160	59886	56402	52125	47568	43282	39784	37501	36706	152
19	62492	479	61714	59465	56022	51793	47290	43053	39596	37339	36554	160
20	7.9962013	497	61244	59024	55622	51447	46998	42811	39399	37168	36394	166
21	61516	514	60758	58566	55207	51086	46694	42562	39194	36993	36228	172
22	61002	535	60254	58090	54778	50712	46381	42304	38980	36810	36056	178
23	60467	555	59728	57597	54334	50325	46053	42038	38761	36622	35878	184
24	59912	573	59184	57086	53871	49923	45717	41761	38534	36426	35694	191
25	59339	589	58623	56556	53391	49505	45371	41474	38298	36226	35503	196
26	58750	605	58046	56014	52901	49080	45009	41180	38055	36016	35307	202
27	58145	621	57453	55457	52396	48640	44639	40876	37806	35802	35105	207
28	57524	636	56843	54882	51878	48190	44261	40565	37550	35584	34898	212
29	56888	650	56221	54297	51348	47727	43874	40246	37288	35358	34686	217
30	7.9956238	662	55584	53653	50819	47257	43478	39920	37008	35128	34469	221
31	55576	674	54935	53089	50253	46775	43074	39589	36748	34893	34248	225
32	54902	687	54274	52465	49691	46288	42662	39253	36470	34655	34023	228
33	54215	698	53601	51831	49119	45793	42244	38907	36189	34412	33795	233
34	53517	711	52917	51183	48536	45283	41819	38559	35909	34167	33562	237
35	52806	717	52220	50532	47943	44768	41385	38202	35606	33915	33325	239
36	52089	724	51518	49869	47346	44248	40949	37844	35313	33661	33086	241
37	51365	732	50809	49202	46742	43723	40508	37482	35015	33405	32845	244
38	50633	740	50090	48527	46132	43193	40060	37116	34715	33147	32601	247
39	49893	744	49365	47845	45515	42655	39610	36745	34409	32884	32354	248
40	7.9949149	749	48637	47146	44893	42115	39156	36372	34099	32652	32106	250
41	48400	754	47902	46468	44269	41572	38700	35997	33793	32357	31856	251
42	47646	757	47164	45772	43640	41026	38241	35620	33484	32090	31605	252
43	46889	756	46422	45074	43009	40477	37779	35242	33173	31823	31353	253
44	46133	761	45681	44377	42378	39928	37319	34862	32862	31556	31100	253
45	45372	757	44934	43676	41744	39377	36856	34483	32547	31287	30847	252
46	44615	759	44193	42977	41114	38828	36394	34102	32236	31019	30595	253
47	43856	758	43449	42276	40483	38278	35931	33723	31925	30750	30342	253
48	43098	754	42707	41578	39849	37727	35468	33344	31612	30483	30089	251
49	42344	748	41968	40883	39221	37181	35011	32967	31302	30217	29838	250
50	7.9941596	746	41232	40192	38596	36639	34555	32593	30995	29952	29588	248
51	40850	742	40503	39505	37975	36100	34101	32220	30687	29688	29340	247
52	40108	734	39777	38820	37357	35560	33648	31848	30383	29427	29093	245
53	39374	726	39057	38143	36745	35028	33201	31481	30081	29166	28848	242
54	38648	720	38347	37474	36139	34504	32759	31118	29782	28910	28606	240
55	37928	708	37641	36810	35539	33980	32320	30758	29485	28655	28366	236
56	37220	701	36947	36158	34938	33466	31888	30403	29193	28405	28130	235
57	36519	691	36258	35508	34362	32956	31459	30051	28905	28155	27895	232
58	35828	676	35579	34869	33784	32455	31036	29700	28618	27910	27663	223
59	35152	665	34918	34251	33226	31969	30629	29368	28344	27674	27440	221
	180° 180°		170° 190°	160° 200°	150° 210°	140° 220°	130° 230°	120° 240°	110° 250°	100° 260°	90° 270°	

TABLE XIX. To convert feet on the Terrestrial Spheroid into Seconds of Arc, and conversely.

Lat.	Log. M. 0° 360°	Diff.	Azimuth from the Meridian, or Z, Log O 10° 350°	20° 340°	30° 330°	40° 320°	50° 310°	60° 300°	70° 290	80° 280°	Log P. 90° 270	D.
60°	7.9934487	652	34268	33638	32669	31468	30223	29037	28070	27439	27219	217
61	33835	638	33631	33036	32127	31013	29826	28711	27802	27209	27002	213
62	33197	623	33004	32447	31596	30551	29438	28391	27539	26984	26789	207
63	32574	608	32394	31874	31076	30099	29059	28080	27283	26764	26582	203
64	31966	591	31797	31312	30569	29658	28689	27776	27033	26549	26379	197
65	31375	572	31221	30769	30071	29231	28329	27481	26790	26339	26182	191
66	30803	557	30659	30240	29600	28816	27980	27194	26553	26136	25991	186
67	30246	539	30111	29726	29135	28412	27640	26917	26325	25939	25805	179
68	29707	520	29584	29230	28687	28022	27316	26646	26103	25749	25626	174
69	29187	499	29074	28750	28252	27644	26996	26386	25889	25565	25452	166
70	7.9928688	478	28587	28290	27832	27283	26693	26135	25684	25389	25286	159
71	28210	458	28118	27851	27440	26939	26403	25898	25489	25222	25127	154
72	27752	437	27669	27426	27057	26604	26121	25668	25297	25057	24973	145
73	27315	417	27240	27024	26693	26288	25856	25450	25118	24904	24828	139
74	26898	392	26831	26640	26345	25985	25602	25241	24946	24756	24689	130
75	26506	368	26448	26278	26020	25703	25363	25045	24786	24619	24559	123
76	26138	344	26087	25939	25712	25436	25140	24861	24635	24488	24436	115
77	25794	322	25750	25622	25425	25186	24930	24688	24494	24366	24321	107
78	25472	296	25435	25325	25158	24953	24734	24528	24362	24254	24214	99
79	25176	273	25145	25052	24910	24739	24553	24379	24239	24147	24115	91
80	24903	250	24877	24799	24682	24541	24387	24244	24127	24048	24024	83
	180° 180°		170° 190°	160° 200°	150° 210°	140° 220°	130° 230°	120° 240°	110° 250°	100° 260°	90° 270°	

TABLE XX. To find the Seconds in the Intercepted Arc reduced for the effect of refraction, as used in the computation of Heights.

Lat	Log M′ 0°	Diff. Lat.	Azimuth from the Meridian Z, Log O′ 10°	20°	30°	40°	50°	60°	70°	80°	Log P′ 90°	Diff. Lat.	Co-lat.
0°	7.619958	131	9871	9620	9235	8762	8258	7784	7398	7145	7058	44	90°
10	9827	377	9743	9499	9126	8667	8179	7719	7345	7099	7014	126	80
20	9450	577	9373	9151	8811	8394	7949	7530	7189	6966	6888	192	70
30	8873	709	8807	8614	8331	7975	7597	7241	6950	6762	6696	236	60
40	8164	755	8113	7964	7738	7461	7165	6886	6659	6514	6460	252	50
50	7409	711	7372	7268	7109	6913	6705	6508	6348	6244	6208	237	40
60	6698	580	6676	6613	6516	6398	6271	6153	6056	5993	5971	193	30
70	6118	379	6108	6078	6032	5977	5918	5863	5817	5788	5778	127	20
80	5739	132	5738	5729	5717	5703	5688	5673	5662	5654	5651	44	10
90	5607		5607	5607	5607	5607	5607	5607	5607	5607	5607		0

TABLE XXI. To compute the Height of the place of Observation by the depression of the horizon of the sea.

Lat.	Log M″ 0°	Azimuth from the Meridian, or Z, Log O″ 10°	20°	30°	40°	50°	60°	70°	80°	Log P″ 90°	Colat.
0°	6.454684	4771	5022	5407	5880	6384	6858	7244	7497	7584	90°
10	4815	4899	5143	5516	5975	6463	6923	7297	7543	7628	80
20	5192	5269	5491	5831	6248	6693	7112	7453	7676	7754	70
30	5769	5835	6028	6311	6667	7045	7401	7692	7880	7946	60
40	6478	6529	6678	6904	7181	7477	7756	7983	8128	8182	50
50	7233	7270	7374	7533	7729	7937	8134	8294	8398	8434	40
60	7944	7963	8029	8126	8244	8371	8489	8586	8649	8671	30
70	8524	8534	8564	8610	8665	8724	8779	8825	8854	8864	20
80	8903	8904	8913	8925	8939	8954	8969	8980	8988	8991	10
90	9035	9035	9035	9035	9035	9035	9035	9035	9035	9035	0
Lat.	180°	170°	160°	150°	140°	130°	120°	110°	100°	90°	Colat.

TABLE XXII. Reduction of λ to *l*. Subtractive.

p''	λ 0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
° ′	″	″	″	″	″	″	″	″	″	° ′ ″
0 0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0 0 0.00
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0 1 0.00
2	0.00	0.01	0.02	0.03	0.04	0.06	0.08	0.13	0.27	0 2 0.00
3	0.00	0.02	0.03	0.05	0.08	0.11	0.16	0.26	0.54	0 3 0.00
4	0.00	0.03	0.05	0.08	0.12	0.17	0.25	0.39	0.81	0 4 0.00
5	0.00	0.04	0.09	0.14	0.20	0.28	0.41	0.65	1.35	0 5 0.00
6	0.00	0.06	0.12	0.19	0.28	0.40	0.58	0.91	1.89	0 6 0.00
7	0.00	0.08	0.16	0.25	0.36	0.54	0.73	1.17	2.43	0 7 0.00
8	0.00	0.10	0.21	0.33	0.48	0.68	0.99	1.57	3.24	0 8 0.00
9	0.00	0.13	0.26	0.41	0.60	0.85	1.23	1.96	4.05	0 9 0.00
0 10	0.00	0.15	0.31	0.49	0.72	1.00	1.48	2.38	4.85	0 10 0.00
11	0.00	0.18	0.38	0.60	0.88	1.20	1.80	2.90	5.80	0 11 0.00
12	0.00	0.22	0.45	0.72	1.05	1.46	2.20	3.40	7.00	0 12 0.00
13	0.00	0.26	0.53	0.86	1.23	1.72	2.60	4.00	8.25	0 13 0.00
14	0.00	0.30	0.62	1.00	1.44	2.00	3.00	4.70	9.55	0 14 0.00
15	0.00	0.34	0.72	1.14	1.66	2.30	3.40	5.40	11.00	0 15 0.00
16	0.00	0.39	0.82	1.30	1.88	2.64	3.87	6.20	12.60	0 16 0.00
17	0.00	0.44	0.92	1.46	2.13	3.00	4.44	7.05	14.25	0 17 0.00
18	0.00	0.50	1.03	1.64	2.38	3.37	5.00	7.90	16.00	0 18 0.00
19	0.00	0.55	1.14	1.82	2.64	3.75	5.50	8.70	17.70	0 19 0.00
0 20	0.00	0.61	1.26	2.01	2.92	4.11	6.00	9.50	19.54	0 20 0.00
21	0.00	0.67	1.40	2.22	3.23	4.55	6.68	10.60	21.70	0 21 0.00
22	0.00	0.74	1.53	2.43	3.56	5.00	7.30	11.70	23.70	0 22 0.00
23	0.00	0.81	1.67	2.65	3.88	5.45	8.00	12.80	26.00	0 23 0.00
24	0.00	0.88	1.83	2.90	4.23	6.00	8.74	13.90	28.40	0 24 0.00
25	0.00	0.96	1.98	3.15	4.58	6.50	9.50	15.00	30.85	0 25 0.00
26	0.00	1.05	2.14	3.40	4.94	7.00	10.28	16.20	33.40	0 26 0.00
27	0.00	1.14	2.32	3.68	5.34	7.55	11.10	17.50	36.00	0 27 0.00
28	0.00	1.22	2.49	3.95	5.74	8.12	11.90	18.80	38.65	0 28 0.00
29	0.00	1.30	2.67	4.25	6.14	8.73	12.70	20.20	41.35	0 29 0.00
0 30	0.00	1.38	2.85	4.53	6.58	9.34	13.60	21.50	44.20	0 30 0.00
31	0.00	1.48	3.04	4.84	7.03	10.00	14.60	23.10	47.50	0 31 0.00
32	0.00	1.58	3.23	5.16	7.47	10.60	15.50	24.50	50.50	0 32 0.00
33	0.00	1.68	3.43	5.50	7.94	11.30	16.50	26.10	53.75	0 33 0.00
34	0.00	1.78	3.65	5.83	8.44	12.00	17.50	27.70	′ 57.00	0 34 0.00
35	0.00	1.88	3.88	6.17	8.94	12.73	18.50	29.40	1 0.50	0 35 0.00
36	0.00	1.99	4.11	6.53	9.48	13.45	19.60	31.10	1 4.00	0 36 0.00
37	0.00	2.11	4.34	6.90	10.00	14.20	20.70	32.90	1 7.75	0 37 0.00
38	0.00	2.23	4.57	7.28	10.55	15.00	21.80	34.60	1 11.25	0 38 0.00
39	0.00	2.35	4.82	7.66	11.12	15.80	23.00	36.40	1 15.00	0 39 0.00
0 40	0.00	2.46	5.09	8.06	11.72	16.62	24.25	38.39	1 19.00	0 40 0.00
41	0.00	2.58	5.35	8.45	12.30	17.47	25.40	40.30	1 23.00	0 41 0.00
42	0.00	2.71	5.60	8.85	12.90	18.34	26.69	42.30	1 27.00	0 42 0.00
43	0.00	2.84	5.88	9.28	13.54	19.20	28.00	44.40	1 31.30	0 43 0.00
44	0.00	2.98	6.15	9.73	14.20	20.10	29.32	46.50	1 35.60	0 44 0.00
45	0.00	3.12	6.43	10.20	14.84	21.05	30.67	48.60	1 40.00	0 45 0.00
46	0.00	3.26	6.72	10.67	15.50	22.00	32.03	50.80	1 44.50	0 46 0.00
47	0.00	3.40	7.02	11.13	16.14	22.95	33.40	53.00	1 49.00	0 47 0.00
48	0.00	3.54	7.32	11.60	16.84	23.94	34.80	55.20	1 53.70	0 48 0.00
49	0.00	3.69	7.63	12.08	17.54	24.96	36.23	57.50	1 58.60	0 49 0.00
0 50	0.00	3.84	7.95	12.58	18.27	26.06	37.65	′ 59.90	2 3.30	0 50 0.00
51	0.00	3.99	8.26	13.10	19.04	27.06	39.30	1 2.40	2 8.40	0 51 0.00
52	0.00	4.17	8.58	13.61	19.80	28.10	40.90	1 4.80	2 13.50	0 52 0.00
53	0.00	4.32	8.92	14.13	20.54	29.17	42.50	1 7.30	2 18.60	0 53 0.00
54	0.00	4.49	9.27	14.67	21.34	30.35	44.10	1 9.90	2 23.80	0 54 0.00
55	0.00	4.66	9.62	15.23	22.14	31.45	45.70	1 12.50	2 29.20	0 55 0.00
56	0.00	4.82	9.97	15.80	22.94	32.58	47.40	1 15.20	2 34.80	0 56 0.00
57	0.00	5.00	10.32	16.35	23.80	33.75	49.10	1 17.90	2 40.40	0 57 0.00
58	0.00	5.18	10.67	16.93	24.64	34.95	50.83	1 20.70	2 46.00	0 58 0.00
59	0.00	5.36	11.05	17.51	25.50	36.20	52.64	1 23.60	2 51.80	0 59 0.00
1 0	0.00	5.54	11.43	18.15	26.34	37.45	′ 54.30	1 26.40	2 57.80	1 0 0.00
1 10	0.00	7.54	15.55	24.68	35.88	′ 50.90	1 13.90	1 57.36	4 1.54	1 10 0.00
1 20	0.00	9.85	20.33	32.25	46.84	1 6.50	1 36.74	2 33.26	5 15.27	1 20 0.00

Log Cor = log p''^2 ½ sin 1″ + log tan λ, log ½ sin 1″ = 4.384545.

Table XXII. Reduction of λ to *l*. Subtractive.

p''		λ									
		50°	51°	52°	53°	54°	55°	56°	57°	58°	59°
°	′	″	″	″	″	″	″	″	″	″	″
0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2	0.06	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.08	0.08
	3	0.11	0.12	0.12	0.13	0.13	0.14	0.14	0.15	0.15	0.16
	4	0.16	0.17	0.18	0.19	0.20	0.20	0.21	0.22	0.23	0.24
	5	0.28	0.30	0.30	0.32	0.33	0.34	0.35	0.37	0.38	0.39
	6	0.40	0.42	0.42	0.44	0.46	0.48	0.49	0.51	0.53	0.55
	7	0.54	0.55	0.55	0.57	0.59	0.61	0.63	0.66	0.68	0.71
	8	0.68	0.71	0.73	0.76	0.78	0.82	0.84	0.88	0.91	0.94
	9	0.85	0.88	0.91	0.95	0.98	1.04	1.07	1.10	1.14	1.18
0	10	1.00	1.08	1.12	1.13	1.20	1.24	1.28	1.32	1.38	1.43
	11	1.20	1.30	1.33	1.35	1.46	1.48	1.56	1.59	1.66	1.71
	12	1.46	1.54	1.58	1.63	1.75	1.75	1.86	1.90	1.98	2.03
	13	1.72	1.81	1.88	1.93	2.06	2.10	2.20	2.26	2.34	2.40
	14	2.00	2.10	2.22	2.25	2.38	2.46	2.56	2.64	2.72	2.80
	15	2.30	2.41	2.58	2.60	2.72	2.82	2.94	3.03	3.12	3.25
	16	2.64	2.75	2.95	2.98	3.08	3.20	3.35	3.45	3.55	3.71
	17	3.00	3.12	3.32	3.37	3.49	3.62	3.77	3.90	4.00	4.19
	18	3.37	3.50	3.70	3.78	3.91	4.05	4.20	4.36	4.48	4.70
	19	3.75	3.88	4.12	4.20	4.34	4.48	4.63	4.83	5.00	5.24
0	20	4.11	4.30	4.52	4.64	4.80	4.95	5.14	5.33	5.53	5.79
	21	4.55	4.74	4.97	5.09	5.30	5.48	5.68	5.87	6.11	6.39
	22	5.00	5.20	5.42	5.58	5.82	6.05	6.25	6.46	6.73	7.00
	23	5.45	5.72	5.91	6.10	6.35	6.62	6.85	7.08	7.38	7.63
	24	6.00	6.24	6.48	6.66	6.90	7.20	7.48	7.73	8.06	8.33
	25	6.50	6.78	7.00	7.25	7.50	7.80	8.12	8.40	8.76	9.05
	26	7.00	7.32	7.54	7.83	8.13	8.43	8.77	9.09	9.48	9.79
	27	7.55	7.88	8.15	8.44	8.78	9.12	9.44	9.80	10.23	10.58
	28	8.12	8.46	8.78	9.07	9.45	9.82	10.16	10.53	10.99	11.39
	29	8.73	9.06	9.42	9.74	10.13	10.54	10.92	11.30	11.76	12.22
0	30	9.34	9.70	10.06	10.40	10.85	11.26	11.69	12.09	12.55	13.07
	31	10.00	10.36	10.74	11.13	11.59	12.00	12.48	12.91	13.38	13.95
	32	10.60	11.03	11.45	11.88	12.35	12.75	13.28	13.77	14.24	14.87
	33	11.30	11.70	12.18	12.63	13.13	13.57	14.09	14.66	15.13	15.81
	34	12.00	12.40	12.93	13.38	13.92	14.41	14.93	15.56	16.05	16.79
	35	12.73	13.14	13.73	14.16	14.72	15.27	15.80	16.48	17.02	17.86
	36	13.45	13.93	14.54	15.00	15.57	16.14	16.71	17.43	18.04	18.93
	37	14.20	14.73	15.36	15.85	16.44	17.07	17.67	18.40	19.09	19.95
	38	15.00	15.55	16.18	16.70	17.34	18.00	18.65	19.41	20.15	21.00
	39	15.80	16.40	17.00	17.57	18.24	18.95	19.63	20.44	21.23	22.08
0	40	16.62	17.25	17.89	18.51	19.20	20.00	20.70	21.51	22.33	23.24
	41	17.47	18.12	18.79	19.47	20.18	21.00	21.77	22.60	23.46	24.41
	42	18.34	19.00	19.73	20.45	21.20	22.00	22.86	23.73	24.61	25.63
	43	19.20	19.93	20.69	21.44	22.24	23.07	23.96	24.88	25.81	26.87
	44	20.10	20.87	21.66	22.45	23.30	24.14	25.08	26.04	27.03	28.12
	45	21.05	21.82	22.64	23.47	24.37	25.27	26.23	27.22	28.27	29.39
	46	22.00	22.82	23.62	24.50	25.46	26.42	27.39	28.42	29.53	30.70
	47	22.95	23.83	24.68	25.58	26.56	27.60	28.57	29.67	30.82	32.05
	48	23.94	24.84	25.76	26.70	27.68	28.78	29.79	30.94	32.15	33.43
	49	24.96	25.85	26.85	27.88	28.83	29.96	31.04	32.22	33.51	34.85
0	50	26.06	26.92	27.93	28.98	30.03	31.20	32.33	33.57	34.88	36.30
	51	27.06	28.01	29.06	30.13	31.26	32.46	33.65	34.95	36.30	37.78
	52	28.10	29.12	30.19	31.32	32.51	33.75	35.00	36.34	37.76	39.27
	53	29.17	30.24	31.36	32.53	33.78	35.05	36.37	37.75	39.23	40.79
	54	30.35	31.41	32.56	33.76	35.07	36.35	37.75	39.20	40.72	42.32
	55	31.45	32.59	33.79	35.03	36.37	37.70	39.14	40.66	42.23	43.90
	56	32.58	33.76	35.04	36.32	37.69	39.07	40.56	42.15	43.76	45.52
	57	33.75	35.00	36.30	37.63	39.04	40.48	42.02	43.66	45.32	47.16
	58	34.95	36.24	37.58	38.96	40.43	41.93	43.50	45.22	46.92	48.84
	59	36.20	37.53	38.90	40.31	41.85	43.41	45.03	46.81	48.57	50.55
1	0	37.45	38.82	40.24	41.76	43.33	44.88	46.63	48.44	50.30	52.32
1	10	50.90	51.82	54.73	56.69	58.85	1 1.07	1 3.35	1 5.80	1 8.38	1 11.08
1	20	1 6.50	1 9.06	1 11.48	1 14.13	1 16.85	1 19.75	1 22.77	1 26.00	1 29.36	1 32.87

TABLE XXIV. To reduce a base at the level of the sea to any height above it, and conversely, &c.

h	mh +	a	pa^2 —	S +	$\Delta_{,}$	Arg.	Eq. Δ_2
Feet.	Correction.	Feet.	Correction.	Reduction.	+		
1000	0.0000208	100000	0.0000004	0.0000008	26	1	0.8
2000	0.0000415	200000	0.0000017	0.0000034	44	2	1.4
3000	0.0000623	300000	0.0000037	0.0000078	60	3	1.8
4000	0.0000830	400000	0.0000065	0.0000138	78	4	2.0
5000	0.0001038	500000	0.0000103	0.0000216	94	5	2.1
6000	0.0001246	600000	0.0000149	0.0000310	112	6	2.0
7000	0.0001453	700000	0.0000203	0.0000422	130	7	1.8
8000	0.0001661	800000	0.0000265	0.0000552	147	8	1.4
9000	0.0001868	900000	0.0000335	0.0000699		9	0.8

To facilitate the calculation of arcs on the terrestrial spheroid, as well as various operations in Geodesy, the following table to $\frac{1}{300}$ of compression has been formed.

TABLE XXV. The measure of one minute of arc at each degree of latitude in English feet.

Latitude.	Minute of Latitude.	Minute of Longitude.	Minute of Perpendic.	Latitude.	Minute of Latitude.	Minute of Longitude.	Minute of Perpendic.
°	Feet.	Feet.	Feet.	°	Feet.	Feet.	Feet.
0	6045.9	6085.7	6085.7	45	6075.7	4310.3	6095.7
1	6045.9	6084.8	6085.7	46	6076.7	4234.7	6096.0
2	6046.0	6082.0	6085.7	47	6077.8	4157.7	6096.4
3	6046.0	6077.4	6085.8	48	6078.8	4079.5	6096.7
4	6046.1	6071.0	6085.8	49	6079.8	4000.0	6097.1
5	6046.3	6062.7	6085.9	50	6080.9	3919.3	6097.4
6	6046.5	6052.6	6085.9	51	6081.9	3837.4	6097.8
7	6046.7	6040.6	6086.0	52	6082.9	3754.4	6098.1
8	6047.0	6026.9	6086.1	53	6083.9	3670.1	6098.4
9	6047.3	6011.3	6086.2	54	6084.9	3584.8	6098.8
10	6047.6	5993.8	6086.3	55	6085.9	3498.3	6099.1
11	6048.0	5974.6	6086.4	56	6086.9	3410.8	6099.4
12	6048.4	5953.6	6086.6	57	6087.9	3322.2	6099.8
13	6048.9	5930.7	6086.7	58	6088.8	3232.5	6100.1
14	6049.3	5906.1	6086.9	59	6089.7	3141.9	6100.4
15	6049.8	5879.6	6087.0	60	6090.7	3050.3	6100.7
16	6050.4	5851.4	6087.2	61	6091.6	2957.8	6101.0
17	6050.9	5821.4	6087.4	62	6092.4	2864.4	6101.3
18	6051.5	5789.7	6087.6	63	6093.3	2770.1	6101.6
19	6052.2	5756.1	6087.8	64	6094.1	2674.9	6101.9
20	6052.8	5720.9	6088.0	65	6095.0	2578.9	6102.1
21	6053.5	5683.9	6088.3	66	6095.7	2482.1	6102.4
22	6054.2	5645.2	6088.5	67	6096.5	2384.5	6102.7
23	6054.9	5604.7	6088.7	68	6097.3	2286.2	6102.9
24	6055.7	5562.6	6089.0	69	6098.0	2187.2	6103.1
25	6056.5	5518.7	6089.3	70	6098.7	2087.5	6103.4
26	6057.3	5473.2	6089.5	71	6099.3	1987.1	6103.6
27	6058.1	5426.1	6089.8	72	6100.0	1886.2	6103.8
28	6059.0	5377.2	6090.1	73	6100.6	1784.6	6104.0
29	6059.8	5326.8	6090.3	74	6101.1	1682.5	6104.2
30	6060.7	5274.7	6090.7	75	6101.7	1579.9	6104.4
31	6061.6	5221.0	6091.0	76	6102.2	1476.8	6104.5
32	6062.6	5165.7	6091.3	77	6102.7	1373.3	6104.7
33	6063.5	5108.9	6091.6	78	6103.1	1269.3	6104.8
34	6064.5	5050.4	6091.9	79	6103.5	1164.9	6105.0
35	6065.4	4990.5	6092.3	80	6103.9	1060.1	6105.1
36	6066.4	4929.0	6092.6	81	6104.2	995.1	6105.2
37	6067.4	4866.0	6092.9	82	6104.6	849.7	6105.3
38	6068.4	4801.6	6093.3	83	6104.8	744.1	6105.4
39	6069.5	4735.5	6093.6	84	6105.1	638.2	6105.5
40	6070.5	4668.2	6093.9	85	6105.3	532.1	6105.6
41	6071.5	4599.4	6094.3	86	6105.4	425.9	6105.6
42	6072.5	4529.2	6094.6	87	6105.6	319.5	6105.7
43	6073.6	4457.6	6095.0	88	6105.6	213.1	6105.7
44	6074.6	4384.6	6095.3	89	6105.7	106.6	6105.7
45	6075.7	4310.3	6095.7	90	6105.7	0.0	6105.7

Table XXVI.
To change mean Solar into Sidereal Time.

Solar Days.	Add	Solar Min.	Add Seconds.	Solar Sec.	Add Parts of a Sec.
	h. m. s.		s.		s.
1	0 3 56.555	1	0.164	1	0.003
2	0 7 53.111	2	0.329	2	0.006
3	0 11 49.666	3	0.493	3	0.008
4	0 15 46.221	4	0.658	4	0.011
5	0 19 42.777	5	0.822	5	0.014
6	0 23 39.332	6	0.986	6	0.017
7	0 27 35.887	7	1.150	7	0.019
8	0 31 32.443	8	1.315	8	0.022
9	0 35 28.998	9	1.479	9	0.025
10	0 39 25.553	10	1.643	10	0.027
11	0 43 22.109	11	1.807	11	0.030
12	0 47 18.664	12	1.972	12	0.033
13	0 51 15.220	13	2.136	13	0.036
14	0 55 11.775	14	2.300	14	0.038
15	0 59 8.330	15	2.464	15	0.041
16	1 3 4.886	16	2.629	16	0.044
17	1 7 1.441	17	2.793	17	0.047
18	1 10 57.996	18	2.957	18	0.050
19	1 14 54.552	19	3.121	19	0.053
20	1 18 51.107	20	3.286	20	0.055
21	1 22 46.662	21	3.450	21	0.058
22	1 26 44.218	22	3.614	22	0.061
23	1 30 40.773	23	3.779	23	0.064
24	1 34 37.328	24	3.943	24	0.066
25	1 38 33.884	25	4.108	25	0.069
26	1 42 30.439	26	4.272	26	0.072
27	1 46 26.994	27	4.436	27	0.075
28	1 50 23.550	28	4.600	28	0.077
29	1 54 20.105	29	4.764	29	0.080
30	1 58 16.660	30	4.928	30	0.082
31	2 2 13.216	31	5.092	31	0.085
32	2 6 9.771	32	5.257	32	0.088
33	2 10 6.326	33	5.421	33	0.091
34	2 14 2.882	34	5.585	34	0.094
35	2 17 59.437	35	5.750	35	0.097
Sol. Hrs.	m. s.	36	5.914	36	0.100
1	0 9.8565	37	6.078	37	0.102
2	0 19.713	38	6.242	38	0.105
3	0 29.569	39	6.407	39	0.107
4	0 39.426	40	6.571	40	0.110
5	0 49.282	41	6.735	41	0.113
6	0 59.139	42	6.900	42	0.116
7	1 8.995	43	7.064	43	0.119
8	1 18.852	44	7.228	44	0.121
9	1 28.708	45	7.393	45	0.124
10	1 38.565	46	7.557	46	0.127
11	1 48.421	47	7.722	47	0.129
12	1 58.278	48	7.886	48	0.132
13	2 8.134	49	8.050	49	0.136
14	2 17.991	50	8.214	50	0.138
15	2 27.847	51	8.378	51	0.141
16	2 37.704	52	8.543	52	0.143
17	2 47.560	53	8.707	53	0.146
18	2 57.416	54	8.872	54	0.149
19	3 7.273	55	9.036	55	0.151
20	3 17.129	56	9.200	56	0.154
21	3 26.986	57	9.364	57	0.157
22	3 36.841	58	9.528	58	0.159
23	3 46.700	59	9.692	59	0.162
24	3 56.555	60	9.856	60	0.164

Acceleration.

Table XXVII.
To change Sidereal into mean Solar Time.

Sidereal Days.	Subtract	Sider. Min.	Subtract Seconds.	Sider. Sec.	Subtr. Pts. of a Sec.
	h. m. s.		s.		s.
1	0 3 55.909	1	0.164	1	0.003
2	0 7 51.819	2	0.328	2	0.005
3	0 11 47.728	3	0.491	3	0.008
4	0 15 43.638	4	0.655	4	0.011
5	0 19 39.547	5	0.819	5	0.014
6	0 23 35.457	6	0.983	6	0.016
7	0 27 31.366	7	1.147	7	0.019
8	0 31 27.276	8	1.311	8	0.022
9	0 35 23.185	9	1.474	9	0.025
10	0 39 19.094	10	1.638	10	0.027
11	0 43 15.004	11	1.802	11	0.030
12	0 47 10.913	12	1.966	12	0.032
13	0 51 6.823	13	2.130	13	0.035
14	0 55 2.732	14	2.294	14	0.038
15	0 58 58.642	15	2.457	15	0.041
16	1 2 54.551	16	2.621	16	0.044
17	1 6 50.461	17	2.785	17	0.046
18	1 10 46.370	18	2.949	18	0.049
19	1 14 42.280	19	3.113	19	0.052
20	1 18 38.189	20	3.277	20	0.055
21	1 22 34.098	21	3.440	21	0.057
22	1 26 30.008	22	3.604	22	0.060
23	1 30 25.917	23	3.768	23	0.063
24	1 34 21.827	24	3.932	24	0.066
25	1 38 17.736	25	4.096	25	0.068
26	1 42 13.646	26	4.259	26	0.071
27	1 46 9.555	27	4.423	27	0.074
28	1 50 5.465	28	4.587	28	0.076
29	1 54 1.374	29	4.751	29	0.079
30	1 57 57.283	30	4.915	30	0.082
31	2 1 53.193	31	5.079	31	0.085
32	2 5 49.102	32	5.242	32	0.087
33	2 9 45.012	33	5.406	33	0.090
34	2 13 40.921	34	5.570	34	0.093
35	2 17 36.831	35	5.734	35	0.096
Sid. Hrs	m. s.	36	5.898	36	0.098
1	0 9.829	37	6.062	37	0.101
2	0 19.659	38	6.225	38	0.104
3	0 29.489	39	6.389	39	0.106
4	0 39.318	40	6.553	40	0.109
5	0 49.148	41	6.717	41	0.112
6	0 58.977	42	6.881	42	0.115
7	1 8.807	43	7.044	43	0.117
8	1 18.636	44	7.208	44	0.120
9	1 28.466	45	7.372	45	0.123
10	1 38.296	46	7.536	46	0.126
11	1 48.125	47	7.699	47	0.128
12	1 57.955	48	7.864	48	0.131
13	2 7.784	49	8.027	49	0.134
14	2 17.614	50	8.191	50	0.137
15	2 27.442	51	8.355	51	0.139
16	2 37.272	52	8.519	52	0.142
17	2 47.103	53	8.683	53	0.145
18	2 56.932	54	8.846	54	0.147
19	3 6.762	55	9.010	55	0.150
20	3 16.591	56	9.174	56	0.153
21	3 26.421	57	9.338	57	0.156
22	3 36.249	58	9.502	58	0.158
23	3 46.080	59	9.666	59	0.161
24	3 55.909	60	9.829	60	0.164

Retardation.

Table XXVIII. Space into Time. To convert Degrees and parts of the Equator into Sidereal Time.

°	h. m.	′	m. s.	″	s.
1	0 4	1	0 4	1	0.066
2	0 8	2	0 8	2	0.133
3	0 12	3	0 12	3	0.200
4	0 16	4	0 16	4	0.266
5	0 20	5	0 20	5	0.333
6	0 24	6	0 24	6	0.400
7	0 28	7	0 28	7	0.466
8	0 32	8	0 32	8	0.533
9	0 36	9	0 36	9	0.600
10	0 40	10	0 40	10	0.666
11	0 44	11	0 44	11	0.733
12	0 48	12	0 48	12	0.799
13	0 52	13	0 52	13	0.866
14	0 56	14	0 56	14	0.933
15	1 0	15	1 0	15	1.000
16	1 4	16	1 4	16	1.066
17	1 8	17	1 8	17	1.133
18	1 12	18	1 12	18	1.200
19	1 16	19	1 16	19	1.266
20	1 20	20	1 20	20	1.333
25	1 40	21	1 24	21	1.400
30	2 0	22	1 28	22	1.466
35	2 20	23	1 32	23	1.533
40	2 40	24	1 36	24	1.600
45	3 0	25	1 40	25	1.666
50	3 20	26	1 44	26	1.733
55	3 40	27	1 48	27	1.799
60	4 0	28	1 52	28	1.866
65	4 20	29	1 56	29	1.933
70	4 40	30	2 0	30	2.000
75	5 0	31	2 4	31	2.066
80	5 20	32	2 8	32	2.133
90	6 0	33	2 12	33	2.200
100	6 40	34	2 16	34	2.266
110	7 20	35	2 20	35	2.333
120	8 0	36	2 24	36	2.400
130	8 40	37	2 28	37	2.466
140	9 20	38	2 32	38	2.533
150	10 0	39	2 36	39	2.600
160	10 40	40	2 40	40	2.666
170	11 20	41	2 44	41	2.733
180	12 0	42	2 48	42	2.799
190	12 40	43	2 52	43	2.866
200	13 20	44	2 56	44	2.933
210	14 0	45	3 0	45	3.000
220	14 40	46	3 4	46	3.065
230	15 20	47	3 8	47	3.133
240	16 0	48	3 12	48	3.200
250	16 40	49	3 16	49	3.266
260	17 20	50	3 20	50	3.333
270	18 0	51	3 24	51	3.400
280	18 40	52	3 28	52	3.466
290	19 20	53	3 32	53	3.533
300	20 0	54	3 36	54	3.600
310	20 40	55	3 40	55	3.666
320	21 20	56	3 44	56	3.733
330	22 0	57	3 48	57	3.799
340	22 40	58	3 52	58	3.866
350	23 20	59	3 56	59	3.933
360	24 0	60	4 0	60	4.000

Or to convert Degrees and parts of Terrestrial Longitude into Time.

Table XXIX. Time into Space. To convert Sidereal Time into Degrees and Parts of the Equator.

h.	°
1	15
2	30
3	45
4	60
5	75
6	90
7	105
8	120
9	135
10	150
11	165
12	180
13	195
14	210
15	225
16	240
17	255
18	270
19	285
20	300
21	315
22	330
23	345
24	360

Tenths.

s.	″
0.1	1.5
0.2	3.0
0.3	4.5
0.4	6.0
0.5	7.5
0.6	9.0
0.7	10.5
0.8	12.0
0.9	13.5
1.0	15.0

Hundredths.

s.	″
0.01	0.15
0.02	0.30
0.03	0.45
0.04	0.60
0.05	0.75
0.06	0.90
0.07	1.05
0.08	1.20
0.09	1.35
0.10	1.50

Thousandths.

s.	″
0.001	0.015
0.002	0.030
0.003	0.045
0.004	0.060
0.005	0.075
0.006	0.090
0.007	0.105
0.008	0.120
0.009	0.135
0.010	0.150

m.	° ′	s.	′ ″
1	0 15	1	0 15
2	0 30	2	0 30
3	0 45	3	0 45
4	1 0	4	1 0
5	1 15	5	1 15
6	1 30	6	1 30
7	1 45	7	1 45
8	2 0	8	2 0
9	2 15	9	2 15
10	2 30	10	2 30
11	2 45	11	2 45
12	3 0	12	3 0
13	3 15	13	3 15
14	3 30	14	3 30
15	3 45	15	3 45
16	4 0	16	4 0
17	4 15	17	4 15
18	4 30	18	4 30
19	4 45	19	4 45
20	5 0	20	5 0
21	5 15	21	5 15
22	5 30	22	5 30
23	5 45	23	5 45
24	6 0	24	6 0
25	6 15	25	6 15
26	6 30	26	6 30
27	6 45	27	6 45
28	7 0	28	7 0
29	7 15	29	7 15
30	7 30	30	7 30
31	7 45	31	7 45
32	8 0	32	8 0
33	8 15	33	8 15
34	8 30	34	8 30
35	8 45	35	8 45
36	9 0	36	9 0
37	9 15	37	9 15
38	9 30	38	9 30
39	9 45	39	9 45
40	10 0	40	10 0
41	10 15	41	10 15
42	10 30	42	10 30
43	10 45	43	10 45
44	11 0	44	11 0
45	11 15	45	11 15
46	11 30	46	11 30
47	11 45	47	11 45
48	12 0	48	12 0
49	12 15	49	12 15
50	12 30	50	12 30
51	12 45	51	12 45
52	13 0	52	13 0
53	13 15	53	13 15
54	13 30	54	13 30
55	13 45	55	13 45
56	14 0	56	14 0
57	14 15	57	14 15
58	14 30	58	14 30
59	14 45	59	14 45
60	15 0	60	15 0

Or to convert Time into Degrees and Parts of Terrestrial Longitude.

Table XXX. Diurnal Variations.

Interv. 24 hrs.	′ m. 10	′ m. 20	′ m. 30	′ m. 1	′ m 2	′ m. 3	′ m. 4	′ m. 5	′ m. 6	′ m. 7	′ m. 8	′ m. 9	Interv. 12 hrs.
h. m.	′ ″ m. s.	′ ″ m. s.	′ ″ m. s.	′ ″ m. s.	′ ″ m. s.	′ ″ m. s.	′ ″ m. s.	′ ″ m. s.	′ ″ m. s.	′ ″ m. s.	′ ″ m. s.	′ ″ m s.	h. m.
0 0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0
0 30	0 12.5	0 25.0	0 37.5	0 1.2	0 2.5	0 3.7	0 5.0	0 6.2	0 7.5	0 8.7	0 10.0	0 11.2	0 15
1 0	0 25.0	0 50.0	1 15.0	0 2.5	0 5.0	0 7.5	0 10.0	0 12.5	0 15.0	0 17.5	0 20.0	0 22.5	0 30
1 30	0 37.5	1 15.0	1 32.5	0 3.7	0 7.5	0 11.2	0 15.0	0 18.7	0 22.5	0 26.2	0 30.0	0 33.7	0 45
2 0	0 50.0	1 40.0	2 30.0	0 5.0	0 10.0	0 15.0	0 20.0	0 25.0	0 30.0	0 35.0	0 40.0	0 45.0	1 0
2 30	1 2.5	2 5.0	3 7.5	0 6.2	0 12.5	0 18.7	0 25.0	0 31.2	0 37.5	0 43.7	0 50.0	0 56.2	1 15
3 0	1 15.0	2 30.0	3 45.0	0 7.5	0 15.0	0 22.5	0 30.0	0 37.5	0 45.0	0 52.5	1 0.0	1 7.5	1 30
3 30	1 27.5	2 55.0	4 22.5	0 8.7	0 17.5	0 26.2	0 35.0	0 43.7	0 52.5	1 1.2	1 10.0	1 18.7	1 45
4 0	1 40.0	3 20.0	5 0.0	0 10.0	0 20.0	0 30.0	0 40.0	0 50.0	1 0.0	1 10.0	1 20.0	1 30.0	2 0
4 30	1 52.5	3 45.0	5 37.5	0 11.2	0 22.5	0 33.7	0 45.0	0 56.2	1 7.5	1 18.7	1 30.0	1 41.2	2 15
5 0	2 5.0	4 10.0	6 15.0	0 12.5	0 25.0	0 37.5	0 50.0	1 2.5	1 15.0	1 27.5	1 40.0	1 52.5	2 30
5 30	2 17.5	4 35.0	6 52.5	0 13.7	0 27.5	0 41.2	0 55.0	1 8.7	1 22.5	1 36.2	1 50.0	2 3.7	2 45
6 0	2 30.0	5 0.0	7 30.0	0 15.0	0 30.0	0 45.0	1 0.0	1 15.0	1 30.0	1 45.0	2 0.0	2 15.0	3 0
6 30	2 42,5	5 25.0	8 7.5	0 16.2	0 32.5	0 48.7	1 5.0	1 21.2	1 37.5	1 53.7	2 10.0	2 26.2	3 15
7 0	2 55.0	5 50.0	8 45.0	0 17.5	0 35.0	0 52.5	1 10.0	1 27.5	1 45.0	2 2.5	2 20.0	2 37.5	3 30
7 30	3 7.5	6 15.0	9 22.5	0 18.7	0 37.5	0 56.2	1 15.0	1 33.7	1 52.5	2 11.2	2 30.0	2 48.7	3 45
8 0	3 20.0	6 40.0	10 0.0	0 20.0	0 40.0	1 0.0	1 20.0	1 40.0	2 0.0	2 20.0	2 40.0	3 0.0	4 0
8 30	3 32.5	7 5.0	10 37.5	0 21.2	0 42.5	1 3.7	1 25.0	1 46.2	2 7.5	2 28.7	2 50.0	3 11.2	4 15
9 0	3 45 0	7 30.0	11 15.0	0 22.5	0 45.0	1 7.5	1 30.0	1 52.5	2 15.0	2 37.5	3 0.0	3 22.5	4 30
9 30	3 57.5	7 55.0	11 52.5	0 23.7	0 47.5	1 11.2	1 35.0	1 58.7	2 22.5	2 46.2	3 10.0	3 33.7	4 45
10 0	4 10.0	8 20.0	12 30.0	0 25.0	0 50.0	1 15.0	1 40.0	2 5.0	2 30.0	2 55.0	3 20.0	3 45.0	5 0
10 30	4 22.5	8 45.0	13 7.5	0 26.2	0 52.5	1 18.7	1 45.0	2 11.2	2 37.5	3 3.7	3 30.0	3 56.2	5 15
11 0	4 35.0	9 10.0	13 45.0	0 27.5	0 55.0	1 22.5	1 50.0	2 17.5	2 45.0	3 12.5	3 40.0	4 7.5	5 30
11 30	4 47.5	9 35.0	14 22.5	0 28.7	0 57.5	1 26.2	1 55.0	2 23.7	2 52.5	3 21.2	3 50.0	4 18.7	5 45
12 0	5 0.0	10 0.0	15 0.0	0 30.0	1 0.0	1 30.0	2 0.0	2 30.0	3 0.0	3 30.0	4 0.0	4 30.0	6 0
12 30	5 12.5	10 25.0	15 37.5	0 31.2	1 2.5	1 33.7	2 5.0	2 36.2	3 7.5	3 38.7	4 10.0	4 41.2	6 15
13 0	5 25.0	10 50.0	16 15.0	0 32.5	1 5.0	1 37.5	2 10.0	2 42.5	3 15.0	3 47.5	4 20.0	4 52.5	6 30
13 30	5 37.5	11 15.0	16 52.5	0 33.7	1 7.5	1 41.2	2 15.0	2 48.7	3 22.5	3 56.2	4 30.0	5 3.7	6 45
14 0	5 50.0	11 40.0	17 30.0	0 35.0	1 10.0	1 45.0	2 20.0	2 55.0	3 30.0	4 5.0	4 40.0	5 15.0	7 0
14 30	6 2.5	12 5.0	18 7.5	0 36.2	1 12.5	1 48.7	2 25.0	3 1.2	3 37.5	4 13.7	4 50.0	5 26.2	7 15
15 0	6.15.0	12 30.0	18 45.0	0 37.5	1 15.0	1 52.5	2 30.0	3 7.5	3 45.0	4 22.5	5 0.0	5 37.5	7 30
15 30	6 27.5	12 55.0	19 22.5	0 38.7	1 17.5	1 56.2	2 35.0	3 13.7	3 52.5	4 31.2	5 10.0	5 48.7	7 45
16 0	6 40.0	13 20.0	20 0.0	0 40.0	1 20.0	2 0.0	2 40.0	3 20.0	4 0.0	4 40.0	5 20.0	6 0.0	8 0
16 30	6 52.5	13 45.0	20 37.5	0 41.2	1 22.5	2 3.7	2 45.0	3 26.2	4 7.5	4 48.7	5 30.0	6 11.2	8 15
17 0	7 5.0	14 10.0	21 15.0	0 42.5	1 25.0	2 7.5	2 50.0	3 32.5	4 15.0	4 57.5	5 40.0	6 22.5	8 30
17 30	7 17.5	14 35.0	21 52.5	0 43.7	1 27.5	2 11.2	2 55.0	3 38.7	4 22.5	5 6.2	5 50.0	6 33.7	8 45
18 0	7 30.0	15 0.0	22 30.0	0 45.0	1 30.0	2 15.0	3 0.0	3 45.0	4 30.0	5 15.0	6 0.0	6 45.0	9 0
18 30	7 42.5	15 25.0	23 7.5	0 46.2	1 32.5	2 18.7	3 5.0	3 51.2	4 37.5	5 23.7	6 10.0	6 56.2	9 15
19 0	7 55.0	15 50.0	23 45.0	0 47.5	1 35.0	2 22.5	3 10.0	3 57.5	4 45.0	5 32.5	6 20.0	7 7.5	9 30
19 30	8 7.5	16 15.0	24 22.5	0 48.7	1 37.5	2 26.2	3 15.0	4 3.7	4 52.5	5 41.2	6 30.0	7 18.7	9 45
20 0	8 20.0	16 40.0	25 0.0	0 50.0	1 40.0	2 30.0	3 20.0	4 10.0	5 0.0	5 50.0	6 40.0	7 30.0	10 0
20 30	8 32.5	17 5.0	25 37.5	0 51.2	1 42.5	2 33.7	3 25.0	4 16.2	5 7.5	5 58.7	6 50.0	7 41.2	10 15
21 0	8 45.0	17 30.0	26 15.0	0 52.5	1 45.0	2 37.5	3 30.0	4 22.5	5 15.0	6 7.5	7 0.0	7 52.5	10 30
21 30	8 57.5	17 55.0	26 52.5	0 53.7	1 47.5	2 41.2	3 35.0	4 28.7	5 22.5	6 16.2	7 10.0	8 3.7	10 45
22 0	9 10.0	18 20.0	27 30.0	0 55.0	1 50.0	2 45.0	3 40.0	4 35.0	5 30.0	6 25.0	7 20.0	8 15.0	11 0
22 30	9 22.5	18 45.0	28 7.5	0 56.2	1 52.5	2 48.7	3 45.0	4 41.2	5 37.5	6 33.7	7 30.0	8 26.2	11 15
23 0	9 35.0	19 10.0	28 45.0	0 57.5	1 55.0	2 52.5	3 50.0	4 47.5	5 45.0	6 42.5	7 40.0	8 37.5	11 30
23 30	9 47.5	19 35.0	29 22.5	0 58.7	1 57.5	2 56.2	3 55.0	4 53.7	5 52.5	6 51.2	7 50.0	8 48.7	11 45
24 0	10 0.0	20 0.0	30 0.0	1 0.0	2 0.0	3 0.0	4 0.0	5 0.0	6 0.0	7 0.0	8 0.0	9 0.0	12 0
m.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	″ s.	m.
2	0.8	1.7	2.5	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.7	1
4	1.7	3.3	5.0	0.2	0.3	0.5	0.7	0.8	1.0	1.2	1.3	1.5	2
6	2.5	5.0	7.5	0.2	0.5	0.7	1.0	1.2	1.5	1.7	2.0	2.2	3
8	3.3	6.7	10.0	0.3	0.7	1.0	1.3	1.7	2.0	2.3	2.7	3.0	4
10	4.2	8.3	12.5	0.4	0.8	1.2	1.7	2.1	2.5	2.9	3.3	3.7	5
12	5.0	10.0	15.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	6
14	5.8	11.7	17.5	0.6	1.2	1.7	2.3	2.9	3.5	4.1	4.7	5.2	7
16	6.7	13.3	20.0	0.7	1.3	2.0	2.7	3.3	4.0	4.7	5.3	6.0	8
18	7.5	15.0	22.5	0.7	1.5	2.2	3.0	3.7	4.5	5.2	6.0	6.7	9
20	8.3	16.7	25.0	0.8	1.7	2.5	3.3	4.2	5.0	5.8	6.7	7.5	10
22	9.2	18.3	27.5	0.9	1.8	2.7	3.7	4.6	5.5	6.4	7.3	8.2	11
24	10.0	20.0	30.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	12
[illegible]	10.8	21.7	32.5	1.1	2.2	3.2	4.3	5.4	6.5	7.6	8.7	9.7	13
	11.7	23.3	35.0	1.2	2.3	3.5	4.7	5.8	7.0	8.2	9.3	10.5	14

TABLE XXXI. Shewing the lengths of *horizontal lines* equivalent to the several ascending and descending planes, the length of the plane being unity; in reference to the different classes of Engines, including the gross load, with engine and tender.

Gradients.	First Class Engines, Load 100 tons. Equivalent Horizontal Planes.			First Class Engines, Load 50 tons. Equivalent Horizontal Planes.			Second Class Engines, Load 80 tons. Equivalent Horizontal Planes.			Second Class Engines, Load 40 tons. Equivalent Horizontal Planes.		
	As.	Dec.	Mean.	As.	Dec.	Mean.	As.	Dec.	Mean.	As.	Dec.	Mean.
1 in 90	2.50	1.00	1.75	1.99	1.00	1.49	2.60	1.00	1.80	2.07	1.00	1.53
95	2.42	1.00	1.71	1.94	1.00	1.47	2.51	1.00	1.75	2.02	1.00	1.51
100	2.39	1.00	1.69	1.89	1.00	1.44	2.44	1.00	1.72	1.97	1.00	1.48
110	2.23	1.00	1.61	1.81	1.00	1.40	2.38	1.00	1.69	1.88	1.00	1.44
120	2.12	1.00	1.56	1.74	1.00	1.37	2.20	1.00	1.60	1.80	1.00	1.40
130	2.04	1.00	1.52	1.68	1.00	1.34	2.10	1.00	1.55	1.74	1.00	1.37
140	1.96	1.00	1.46	1.64	1.00	1.32	2.03	1.00	1.51	1.69	1.00	1.34
160	1.84	0.83	1.33	1.56	0.83	1.20	1.90	0.83	1.36	1.60	0.83	1.21
180	1.79	0.83	1.31	1.49	0.83	1.16	1.80	0.83	1.31	1.53	0.83	1.18
200	1.67	0.83	1.25	1.44	0.83	1.13	1.72	0.83	1.27	1.48	0.83	1.15
250	1.53	0.83	1.18	1.36	0.83	1.09	1.58	0.83	1.20	1.42	0.83	1.12
300	1.45	0.83	1.14	1.30	0.83	1.06	1.48	0.83	1.15	1.32	0.83	1.07
350	1.38	0.83	1.10	1.25	0.83	1.04	1.41	0.83	1.12	1.27	0.83	1.05
400	1.33	0.83	1.08	1.22	0.83	1.02	1.36	0.83	1.09	1.24	0.83	1.03
500	1.27	0.83	1.05	1.18	0.83	1.01	1.28	0.83	1.05	1.19	0.83	1.01
750	1.18	0.83	1.01	1.12	0.88	1.00	1.19	0.83	1.01	1.13	0.88	1.00
1000	1.13	0.85	1.00	1.09	0.91	1.00	1.14	0.86	1.00	1.09	0.91	1.00
1500	1.09	0.90	1.00	1.06	0.94	1.00	1.09	0.91	1.00	1.03	0.94	1.00

Gradients.	Third Class Engines, Load 80 tons. Equivalent Horizontal Planes.			Third Class Engines, Load 40 tons. Equivalent Horizontal Planes.			Fourth Class Engines Load 60 tons. Equivalent Horizontal Planes.			Fourth Class Engines, Load 30 tons. Equivalent Horizontal Planes.		
	As.	Dec.	Mean.	As.	Dec.	Mean.	As.	Dec.	Mean.	As.	Dec.	Mean.
1 in 90	2.66	1.00	1.83	2.14	1.00	1.57	2.51	1.00	1.75	2.00	1.00	1.50
95	2.58	1.00	1.79	2.08	1.00	1.54	2.44	1.00	1.72	1.95	1.00	1.47
100	2.50	1.00	1.75	2.02	1.00	1.51	2.36	1.00	1.68	1.90	1.00	1.45
110	2.36	1.00	1.68	1.93	1.00	1.46	2.33	1.00	1.66	1.82	1.00	1.41
120	2.25	1.00	1.62	1.85	1.00	1.42	2.14	1.00	1.57	1.75	1.00	1.37
130	2.15	1.00	1.57	1.78	1.00	1.39	2.05	1.00	1.52	1.69	1.00	1.34
140	2.07	1.00	1.53	1.73	1.00	1.36	1.97	1.00	1.48	1.64	1.00	1.32
160	1.94	0.83	1.43	1.64	0.83	1.23	1.85	0.83	1.34	1.56	0.83	1.20
180	1.83	0.83	1.33	1.57	0.83	1.20	1.75	0.83	1.29	1.50	0.83	1.16
200	1.75	0.83	1.29	1.52	0.83	1.17	1.68	0.83	1.25	1.45	0.83	1.14
250	1.60	0.83	1.21	1.41	0.83	1.12	1.54	0.83	1.18	1.35	0.83	1.09
300	1.50	0.83	1.16	1.34	0.83	1.08	1.45	0.83	1.14	1.30	0.83	1.06
350	1.43	0.83	1.13	1.29	0.83	1.06	1.39	0.83	1.10	1.26	0.83	1.04
400	1.37	0.83	1.10	1.25	0.83	1.04	1.34	0.83	1.08	1.22	0.83	1.02
500	1.30	0.83	1.06	1.20	0.83	1.01	1.23	0.83	1.03	1.18	0.83	1.01
750	1.20	0.83	1.01	1.13	0.87	1.00	1.18	0.83	1.01	1.12	0.83	1.00
1000	1.15	0.85	1.00	1.10	0.90	1.00	1.13	0.87	1.00	1.09	0.91	1.00
1500	1.10	0.90	1.00	1.07	0.93	1.00	1.09	0.91	1.00	1.06	0.94	1.00

Gradients.	Mean Class Engines, Load 50 tons. By W. G. Equivalent Horizontal Planes.			Gradients.	Equivalent Horizontal Planes.			Gradients.	Velocities in Miles an Hour. By Dr Lardner. Ascen. Plane.	Dec. Plane.	Mean or Level.
	As.	Dec.	Mean.		As.	Dec.	Mean.		Miles.	Miles.	Miles.
1 in 200	1.28	0.72	1.00	1 in 500	1.12	0.88	1.00	1 in 177	22.2[illegible]	41.32	31.78
250	1.25	0.75	1.00	550	1.10	0.90	1.00	265	24.87	39.13	32.00
300	1.22	0.78	1.00	600	1.08	0.92	1.00	330	25.26	37.07	31.16
350	1.20	0.80	1.00	650	1.05	0.95	1.00	400	26.87	36.75	31.81
400	1.17	0.83	1.00	700	1.03	0.97	1.00	532	27.35	34.30	30.82
450	1.15	0.85	1.00	Level	1.00	1.00	1.00	590	27.37	33.16	30.26
								650	29.03	32.58	30.80
								Level or Mean .			31.23

TABLE XXXII. Computation of Cuttings and Embankments, the formation-level or base being 30 feet, and length one chain.

	Slopes, 1 to 1.			Slopes, 1½ to 1.			Slopes, 2 to 1.		
Depth of Cutting in Feet.	Half width at top in Feet.	Content in cubic yards per chain.	Content of 1 perpendicular foot in breadth.	Half width at top in Feet.	Content in cubic yards per chain.	Content of 1 perpendicular foot in breadth.	Half width at top in Feet.	Content in cubic yards per chain.	Content of 1 perpendicular foot in breadth.
1	16	75.78	2.44	16.5	77.00	2.44	17	78.22	2.44
2	17	156.42	4.89	18.0	161.33	4.89	19	166.22	4.89
3	18	242.00	7.33	19.5	253.00	7.33	21	264.00	7.33
4	19	332.44	9.78	21.0	352.00	9.78	23	371.55	9.78
5	20	427.78	12.22	22.5	453.33	12.22	25	488.89	12.22
6	21	528.00	14.67	24.0	572.00	14.67	27	616.00	14.67
7	22	633.11	17.11	25.5	693.00	17.11	29	752.89	17.11
8	23	743.11	19.56	27.0	821.33	19.56	31	899.55	19.56
9	24	858.00	22.00	28.5	957.00	22.00	33	1056.00	22.00
10	25	977.78	24.44	30.0	1100.00	24.44	35	1222.22	24.44
11	26	1102.44	26.89	31.5	1250.33	26.89	37	1398.22	26.89
12	27	1232.00	29.33	33.0	1408.00	29.33	39	1584.00	29.33
13	28	1366.44	31.78	34.5	1573.00	31.78	41	1779.55	31.78
14	29	1505.78	34.22	36.0	1745.33	34.22	43	1984.89	34.22
15	30	1650.00	36.66	37.5	1925.00	36.66	45	2200.00	36.66
16	31	1799.11	39.11	39.0	2112.00	39.11	47	2424.89	39.11
17	32	1953.11	41.55	40.5	2306.33	41.55	49	2659.55	41.55
18	33	2112.00	43.99	42.0	2508.00	43.99	51	2904.00	43.99
19	34	2275.78	46.44	43.5	2717.00	46.44	53	3158.22	46.44
20	35	2444.44	48.89	45.0	2933.33	48.89	55	3422.22	48.89
21	36	2618.00	51.33	46.5	3517.00	51.33	57	3696.00	51.33
22	37	2793.44	53.77	48.0	3388.00	53.77	59	3979.55	53.77
23	38	2979.78	56.21	49.5	3626.33	56.21	61	4272.89	56.21
24	39	3168.00	58.66	51.0	3872.00	58.66	63	4576.00	58.66
25	40	3331.11	61.10	52.5	4125.00	61.10	65	4888.89	61.10
26	41	3599.11	63.55	54.0	4385.33	63.55	67	5211.55	63.55
27	42	3762.00	65.99	55.5	4653.00	65.99	69	5544.00	65.99
28	43	3969.78	68.43	57.0	4928.00	68.43	71	5886.22	68.43
29	44	4182.44	70.88	58.5	5210.33	70.88	73	6238.22	70.88
30	45	4400.00	73.32	60.0	5500.00	73.32	75	6600.00	73.22
31	46	4622.44	75.77	61.5	5797.00	75.77	77	6971.55	75.77
32	47	4849.78	78.22	63.0	6101.33	78.22	79	7352.89	78.22
33	48	5082.00	80.67	64.5	6413.00	80.67	81	7744.00	80.67
34	49	5319.11	83.11	66.0	6732.00	83.11	83	8144.89	83.11
35	50	5561.11	85.55	67.5	7058.33	85.55	85	8555.55	85.55
36	51	5808.00	88.00	69.0	7392.00	88.00	87	8976.00	88.00
37	52	6059.78	90.44	70.5	7733.00	90.44	89	9406.22	90.44
38	53	6316.44	92.39	72.0	8081.33	92.39	91	9846.22	92.39
39	54	6578.00	95.33	73.5	8437.00	95.33	93	10296.00	95.33
40	55	6844.44	97.77	75.0	8800.00	97.77	95	10755.55	97.77
41	56	7115.78	100.22	76.5	9170.33	100.22	97	11224.89	100.22
42	57	7392.00	102.66	78.0	9548.00	102.66	99	11704.00	102.66
43	58	7673.11	105.11	79.5	9933.00	105.11	101	12192.89	105.11
44	59	7959.11	107.55	81.0	10325.33	107.55	103	12691.55	107.55
45	60	8250.00	109.99	82.5	10725.00	109.99	105	13200.00	109.99
46	61	8545.78	112.44	84.0	11132.00	112.44	107	13718.22	112.44
47	62	8846.44	114.88	85.5	11546.33	114.88	109	14246.22	114.88
48	63	9152.00	117.33	87.0	11968.00	117.33	111	14784.00	117.33
49	64	9462.44	119.77	88.5	12397.00	119.77	113	15331.55	119.77
50	65	9777.78	122.21	90.0	12833.33	122.21	115	15888.89	122.21

EXAMPLE of the use of this Table. If a cutting of one Imperial Chain of 100 links in length and 20 feet in depth, were executed on a base or formation-level of 30 feet, how many cubic yards of earth would be thrown out, the slopes being 2 to 1?

To depth 20 feet in the left hand column, and under slopes 2 to 1 at top, there will be found 3422.22 cubic yards. Mr Macneill's Tables give 51.85 × 66 = 3422.10 cubic yards.